Math 220–01 (Kunkle), Final Exam. Name: ______________________

200 pts, 3 hours Apr. 29, 2011 Page 1

Read each problem carefully. No calculators, notes, books, or any outside materials. Unless a problem specifically refers to degrees, use radians in all questions and answers involving angles or trig functions. You are expected to know the values of all trigonometric functions at multiples of $\pi/4$ and of $\pi/6$. If you wish, you can leave unfinished arithmetic in your final answers.

Unless otherwise indicated, **supporting work will be required on every problem**; one-word answers, or answers which simply restate the question, will receive no credit.

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1 (12 pts). Graph the polar equation $r = 1 - 2\sin\theta$.
List all the θs in $[0, 2\pi]$ at which r is maximized, minimized, or is zero. Plot these points accurately in your graph. Note that the rays through the origin in this graph paper have been drawn at specific values of θ. Use them correctly.

2 (10 pts). Find the area enclosed by one loop of the graph of $r = 3\cos 2\theta$. (You are not required to graph the curve, but use the graph paper on back if you find it helpful.)

3 (6 pts). Find $\frac{dy}{dx}$ along the graph of $r = 3\cos 2\theta$.

4 (14 pts). This problem concerns the curve given parametrically by

$$x = t^2 + t + 2 \text{ and } y = t^2 - t - 3.$$

(Work below, but label your answers a., b., c., d., e., and f.)

a. Find $\frac{dy}{dx}$ along this curve.

b. Find an equation of the line tangent to this curve at the point corresponding to $t = 0$.

c. At what time(s) t, if any, does this curve have a horizontal tangent?

d. At what time(s) t, if any, does this curve have a vertical tangent?

e. Find $\frac{d^2y}{dx^2}$ along this curve.

f. On what interval(s) of t is the graph concave up?

5 (12 pts). Find the length of the segment of the curve given by the parametric equations $x = e^t \sin t$ and $y = e^t \cos t$ for $0 \le t \le \frac{\pi}{2}$.

6 (4 pts). The segment in Problem 5 is rotated about the x-axis. Find the area of the resulting surface. Express your answer as a definite integral, but **do not evaluate**.

7 (4 pts). Find a Cartesian equation for the curve given by the polar equation $r = 6\cos\theta$.

8 (12 pts). Find the average value of the function $\tan^{-1} x$ on the interval $[0, 1]$.

9 (17 pts). Integrate: $\int \frac{1}{(9-x^2)^{5/2}}\, dx$

10 (9 pts). Integrate: $\int \frac{6x^2+x+2}{3x+2}\, dx$

11 (10 pts). Evaluate the limits.

a. $\displaystyle\lim_{n\to\infty} \frac{e^n - 3e^{-n}}{3e^n + e^{-n}}$ b. $\displaystyle\lim_{n\to-\infty} \frac{e^n - 3e^{-n}}{3e^n + e^{-n}}$

12 (6 pts). Evaluate the improper integral, if it exists: $\int_0^3 \frac{2}{(x-1)^2}\, dx$

13. Let R be the region in the xy-plane that lies above the curve $y = x^2$ and below the curve $y = 4 - x^2$.

a (2 pts). Sketch this region. Include the x- and y-axes in your drawing.

b (8 pts). R is rotated about the line $y = -2$. Find the volume of the resulting solid. Express your answer as a definite integral but **do not integrate**.

14 (12 pts). Determine whether the series converges or diverges: $\displaystyle\sum_{n=1}^{\infty} \frac{2+\sqrt{n}}{\sqrt{2n^8+3n^3}}$

15 (16 pts). Find the interval of convergence for the series: $\displaystyle\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n^2 3^n}$

16a (7 pts). Find the general solution to $\frac{dy}{dx} = x\cos y$.

16b (3 pts). Find the particular solution to $\frac{dy}{dx} = x\cos y$ that passes through the point $x = 1$, $y = 0$.

17a (8 pts). Find a formula for the nth partial sum of the series: $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$

17b (2 pts). Find the sum of the series in problem 17a, if it exists.

18. Find the Maclaurin series for each of the functions. Do not use binomial coefficients $\binom{k}{n}$ in part c.

a (3 pts). e^x b (7 pts). $\tan^{-1} x^2$ c (7 pts). $(2-x)^3 - (1+x)^2$

19 (9 pts). If $f(x) = e^x$ and $T_3(x)$ is its third-degree Taylor polynomial centered at $x = 0$, find an upper bound for $|f(x) - T_3(x)|$ on the interval $[-0.6, 0.6]$.
Leave unfinished arithmetic in your answer. You are not required to find $T_3(x)$.

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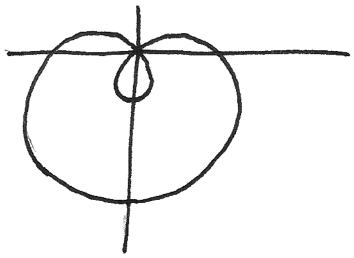
1. $r = 1 - 2\sin\theta$



goes through $(1,0)$ $(0, 5\pi/6)$ $(0, \pi/6)$ $(1, \pi)$ $(-1, \pi/2)$ $(3, \frac{3\pi}{2})$ $(1, 2\pi)$

2. $\frac{9\pi}{8}$ 3. $\frac{dy}{dx} = \frac{-2\sin 2\theta \sin\theta + \cos 2\theta \cos\theta}{-2\sin 2\theta \cos\theta - \cos 2\theta \sin\theta}$

4.a. $\frac{dy}{dx} = \frac{2t-1}{2t+1}$ b. $y+3 = -(x-2)$ c. $t = 1/2$ d. $t = -1/2$.

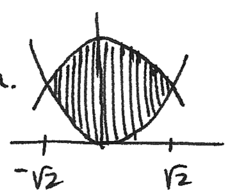
e $\frac{d^2y}{dx^2} = \frac{4}{(2t+1)^3}$ f. $t > -1/2$

5. $\sqrt{2}\,(e^{\pi/2} - 1)$ 6. $2\sqrt{2}\pi \int_0^{\pi/2} e^{2t} \cos t \, dt$ 7. $x^2+y^2 = 6x$

8. $\frac{\pi}{4} - \frac{1}{2}\ln 2$ 9. $3^{-4}\left(\frac{x}{\sqrt{9-x^2}} + \frac{1}{3}\left(\frac{x}{\sqrt{9-x^2}}\right)^3\right) + C$

10. $x^2 - x + \frac{4}{3}\ln|3x+2| + C$ 11.a. $\frac{1}{3}$ b. -3 12. Diverges

13.a.



b. $\pi \int_{-\sqrt{2}}^{\sqrt{2}} \left((6-x^2)^2 - (x^2+2)^2\right) dx$

14. Converges 16. $[-2, 1]$

16a. $\ln|\sec y + \tan y| = \frac{1}{2}x^2 + C$. 16b. $\ln|\sec y + \tan y| = \frac{1}{2}x^2 - \frac{1}{2}$

17a. $S_n = \ln(n+1)$ 17b. Diverges 18a. $\sum_{n=0}^{\infty} \frac{1}{n!} x^n$ 18b. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)}$

18c. $7 - 14x + 5x^2 - x^3$ 19. $|R_3| \le \frac{e^{0.6}(0.6)^4}{4!}$