MATHEMATICS 120 COMMON FINAL EXAM MONDAY DECEMBER 9, 2002 NON-CALCULATOR PART

Hand this part in before getting your calculator

- Give **solutions**, not just **answers**: provide adequate working or explanation, and **circle your answer**.
- The point total is 200.

(1) (20 points) Find the limit, the infinite limit, or state that it does not exist. **Show your work or reasoning.**

(a) $\lim_{x\to 3} \frac{x-3}{x^2-9}$

(b) $\lim_{x\to 0} \frac{\sin(3x)}{4x}$

(c) $\lim_{x\to -2} \frac{3-\sqrt{x+11}}{x+2}$

(d) Given that $\lim_{x\to 2} f(x) = 7$, $\lim_{x\to 2} g(x) = -4$, find $\lim_{x\to 2} [3xf(x) - 5g(x)]$

(e) $\lim_{x\to\infty} \frac{2+\ln(x^3)}{7+\ln(x^5)}$

(2) (10 points) Use the **definition of derivative** in terms of limits to find $f'(2)$ where

$$f(x) = \frac{1}{x+1}.$$

(You may **check** your answer by using the **quotient rule, power rule, etc., but to receive credit for this question you must use the definition of derivative and evaluate the limit algebraically.**)

(3) (27 points) In this question you will differentiate and identify the graphs of the three functions

$$p(x) = -x^7 - x^2 - 5x \qquad q(x) = xe^{2x} \qquad r(x) = -1 + \sqrt{x+1}.$$

(a) Write the first derivative of each function:

$p'(x) =$

$q'(x) =$

$r'(x) =$

(b) Write the second derivative of each function:

$p''(x) =$

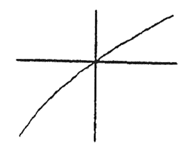
$q''(x) =$

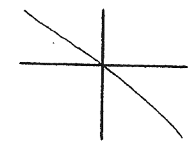
$r''(x) =$

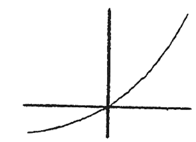
(c) Evaluate your answers at $x = 0$:

p	q	r
$p'(0) =$	$q'(0) =$	$r'(0) =$
$p''(0) =$	$q''(0) =$	$r''(0) =$

(d) The graphs of the functions p, q, and r near $x = 0$ are shown below. **Use your answer from part (c) to match the functions with their graphs. Just write the name of the function ("p", "q", "r") on the appropriate graph.**







(4) (6 points) The temperature, T, in degrees Fahrenheit, of a potato in a hot oven is given by $T = f(t)$, where t is the time in minutes since the potato was put in the oven.

(a) What are the units of $f'(20)$?

(b) What is the practical meaning of the statement $f'(20) = 2$?

(5) (12 points)

(a) Find $\frac{dy}{dx}$ if $y = \ln(3x^2+5) + \sin(e^{2x}+2)$. (Do not simplify your answer.)

(b) Find the derivative of the function $f(x) = \frac{x}{x^2+4}$.

(c) Evaluate $\frac{d}{dx}\left[\int_1^{x^2} \ln\left(1+\sqrt{t}\right)\, dt\right]$.

(6) (13 points) Find the equation of the tangent line to the curve

$$x^2\sqrt{y-2} = y^2 - 3x + y - 8$$

at the point $(1, 3)$. (You may leave the equation in point-slope form.)

(7) (20 points) For the function $f(x) = 3x^4 - 8x^3 + 6x^2 - 3$:

(a) Find all of the critical points.

(b) State clearly where the function is increasing or decreasing: give intervals or use a sign diagram.

(c) Find all the maximum and minimum points, giving their x coordinates.

(d) State clearly where the function is concave up and where it is concave down: give intervals or use a sign diagram.

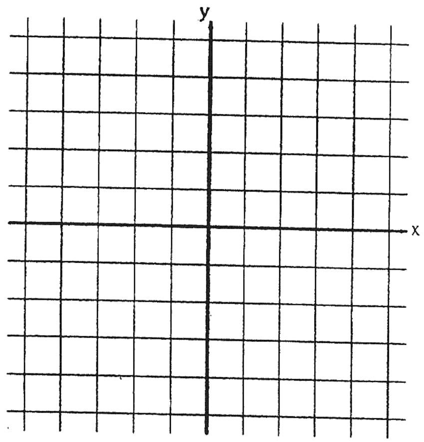
(e) Find all the x values at which f has a point of inflection.

Note: it is not necessary to sketch the graph.

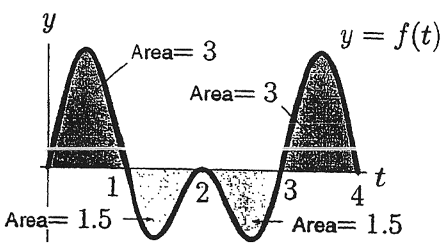
(8) (10 points) Find the absolute maximum and absolute minimum values of $f(x) = x^2 + \frac{16}{x}$ on the interval $[1, 8]$.

(9) (10 points) Sketch a possible graph of a function f that has all the following properties:

a: f is everywhere continuous;
b: $f(-1) = -2$, $f(0) = 0$
c: f' does not exist at $x = -1$;
d: $f'(x) < 0$ for $x < -1$ or $x > 0$, $f'(x) > 0$ for $-1 < x < 0$;
e: $f''(x) < 0$ for $x < -1$ or $-1 < x < 1$, $f''(x) > 0$ for $x > 1$;
f: $\lim_{x\to-\infty} f(x) = 1$, $\lim_{x\to\infty} f(x) = -2$.



(10) (10 points) The function $f(t)$ is shown in the graph below. Note that the two regions above the t-axis each have area 3 while the two regions below the axis each have area 1.5. Use this information to answer the following questions about the function $g(x)$ defined by the formula $g(x) = \int_0^x f(t)\, dt$.



(a) Evaluate $g(1)$ and $g(4)$.

(b) What happens on the graph of $g(x)$ at $x = 3$? (Circle one)

(i) g has a "sharp corner" there
(ii) g has a local min there
(iii) g has a local max there
(iv) g has an inflection point there
(v) None of the above

(c) From this graph we can tell that $g'(3.5)$ is (circle one):

(i) positive
(ii) negative
(iii) zero
(iv) undefined

(11) (12 points) Evaluate each of the following definite integrals. You **must** show at least one intermediate step in your computation to get credit. You do not need to simplify your answers.

(a) $\int_1^2 (x^3 - 2)\, dx$

(b) $\int_0^{\frac{\pi}{2}} \cos(x)\left[\sin^2(x) - 2\sin(x) + 1\right] dx$

(c) $\int_0^2 |(x-1)(x-4)|\, dx$

(12) (20 points) Find the most general indefinite integral. If you use a substitution, show your work. You do not need to simplify your answers.

(a) $\int \left(7 + 3\cos(t)\right) dt$

(b) $\int \left(t^{3/5} - \frac{1}{t}\right) dt$

(c) $\int \frac{5x^4+1}{x^2} dx$

(d) $\int \frac{x}{x^2-1}\, dx$

(e) $\int x\sqrt{4x-5}\, dx$

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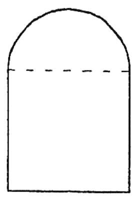
(13) (10 points) A spherical balloon is being inflated, with air entering at a rate of 30 cubic inches per second. When the radius is 10 inches, how fast is the radius increasing?

(14) (10 points) A Norman window has a semicircular upper section sitting above a rectangular part of the same width, as shown. You wish to construct such a window of total area 20 square feet and shortest perimeter: what dimensions should it have?



(15) (10 points)

(a) Approximate the value of the definite integral

$$\int_0^6 \frac{1}{1+t^2}\,dt$$

by a Riemann sum using right endpoints and $n = 3$ sub-intervals.

(b) Based on the fact that the function $f(t) = 1/(1+t^2)$ is a **decreasing** function on the interval $[0, 6]$, how would you say your answer in part (a) compares to the actual value of the integral?

(c) If $f(t) = 1/(1+t^2)$ is the rate (in gallons/minute) at which water is flowing in to a tank at t minutes, what does the value of the integral in part (a) tell you?

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(1) a.) $\frac{1}{6}$ b.) $\frac{3}{4}$ c.) $-\frac{1}{6}$ d.) 62 e.) $\frac{3}{5}$

(2) $f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \ldots = -\frac{1}{9}$

(3) a.) $p'(x) = -7x^6 - 2x - 5$ b.) $p''(x) = -42x^5 - 2$ c.) $p'(0) = -5$; $q'(0) = 1$; $r'(0) = \frac{1}{2}$

$q'(x) = e^{2x} + 2xe^{2x}$ $q''(x) = 4e^{2x} + 4xe^{2x}$ $p''(0) = -2$; $q''(0) = 4$; $r''(0) = -\frac{1}{4}$

$r'(x) = \frac{1}{2}(x+1)^{-1/2}$ $r''(x) = -\frac{1}{4}(x+1)^{-3/2}$

d.) left graph is r ; middle graph is p ; right graph is q

(4) a.) °F/min b.) The temperature of the potato is rising at a rate of 2°F/min at time 20 minutes after it was put in the oven.

(5) a.) $y' = \frac{6x}{3x^2+5} + 2e^{2x}\cos(e^{2x}+2)$ b.) $f'(x) = \frac{4-x^2}{(x^2+4)^2}$

c.) $2x\ln(1+\sqrt{x^2})$ (6) $y - 3 = \frac{10}{13}(x-1)$

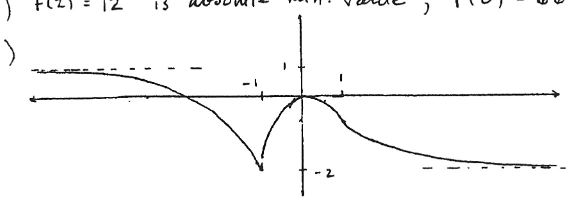
(7) a.) $x = 0$, $x = 1$ b.) $f(x)$ is decreasing on $(-\infty, 0)$; increasing $(0,1)$ & $(1,\infty)$.

c.) Local min. at $x = 0$. d.) $f(x)$ concave up $(-\infty, \frac{1}{3})$ & $(1,\infty)$; concave down $(\frac{1}{3}, 1)$.

e.) Points of inflection at $x = \frac{1}{3}$, $x = 1$.

(8) $f(2) = 12$ is absolute min. value ; $f(8) = 66$ is absolute max. value.

(9)



(10) a.) $g(1) = 3$; $g(4) = 3$

b.) (ii)

c.) (i)

(11) a.) $\frac{7}{4}$ b.) $\frac{1}{3}$ c.) 3

(12) a.) $7t + 3\sin(t) + C$ b.) $\frac{5}{8}t^{8/5} - \ln|t| + C$ c.) $\frac{5}{3}x^3 - x^{-1} + C$

d.) $\frac{1}{2}\ln|x^2 - 1| + C$ e.) $\frac{1}{16}\left(\frac{2}{5}(4x-5)^{5/2} + \frac{10}{3}(4x-5)^{3/2}\right) + C$

(13) $\frac{dr}{dt} = \frac{3}{40\pi} \approx .0239$ inches/sec. (14) Dimensions: $x \approx 4.7$ ft , $y \approx 2.4$ ft

(15) a.) Riemann sum ≈ .57 b.) The Riemann sum in part a. is an underestimate.

c.) $\int_0^6 f(t)\,dt$ = total change, and therefore from time t = 0 min. to t = 6 min. the volume of water in the tank increases by $\int_0^6 f(t)\,dt$ gallons.