

Math 105 - Final Exam Review

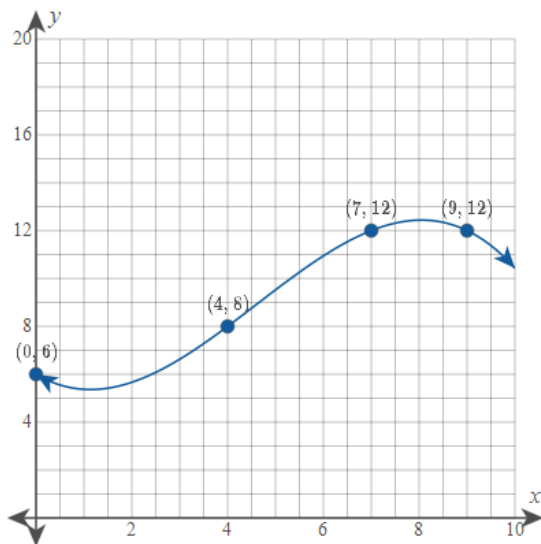
1. Find the limit algebraically by factoring the expression first.

$$\lim_{x \rightarrow 8} \left( \frac{7x^2 - 448}{x - 8} \right)$$

2. Find the limit algebraically by factoring the expression first.

$$\lim_{x \rightarrow 1} \left( \frac{x-1}{9x^2-9} \right)$$

3. Consider the graph of  $f(x)$ .



What is the average rate of change of  $f(x)$  from  $x_1 = 0$  to  $x_2 = 4$ ? Please write your answer as an integer or simplified fraction.

4. What is the average rate of change of  $f(x)$  from  $x_1 = 3$  to  $x_2 = 7.5$ ? Please write your answer rounded to the nearest hundredth.

$$f(x) = 2x^2 - 6x + 9$$

5. Suppose  $f(x)$  models the total number of people that have rented a certain movie, and  $x$  is the number of days since it came out.

**Step 1.** Interpret the meaning of  $f(10) = 700$ .

**Step 2.** Interpret the meaning of  $f'(10) = 3$ .

6. The number of strikeouts per game in Major League Baseball can be approximated by the function  $f(x) = 0.065x + 5.09$ , where  $x$  is the number of years after 1977 and corresponds to one year of play.

**Step 1.** What is the value of  $f(5)$  and what does it represent?

**Step 2.** What is the value of  $f'(5)$  and what does it represent?

7. Find the derivative for the following function.

$$g(x) = \frac{-7}{x}$$

8. Find the derivative for the following function.

$$f(x) = -6\sqrt[4]{x}$$

9. Find the derivative for

$$f(x) = -4x^2 + 2x^3$$

10. Find the derivative for

$$g(x) = 5x^{\frac{5}{2}} + 2 - 3x^{\frac{3}{2}}$$

11. Use algebraic techniques to rewrite  $g(x) = (6x^2 + 5)(2x^2 + 2)$  as a sum or difference; then find  $g'(x)$ .

12. For the function  $f(x) = -7x^3 - 8x + 2x^2$ ,

**Step 1.** Find the slope of the tangent line at  $x = 1$ .

**Step 2.** Find the equation of the tangent line at  $x = 1$ .

13. When a factory operates from 6 AM to 6 PM, its total fuel consumption varies according to the formula  $f(t) = 0.4t^2 - 0.2t^{0.3} + 20$ , where  $t$  is the time in hours after 6 AM and  $f(t)$  is the number of barrels of fuel oil.

**Step 1.** How much fuel is consumed by 9 AM? Round your answer to 2 decimal places.

**Step 2.** What is the rate of consumption of fuel at 11 AM? Round your answer to 2 decimal places.

**Step 3.** What is the average rate of consumption from 6 AM to 1 PM? Round your answer to 2 decimal places.

14. A hotdog vendor has initial start up costs of \$7300 and an item charge of \$0.50 per combo meal. What is the cost function?

15. The weekly cost of producing  $x$  hats is given by the function  $C(x) = 1600 + 38x + 0.3x^2$ . Find the marginal average cost function.

16. A sales representative for a company that produces blenders can sell  $x$  units of their deluxe model if the price is  $p = D(x) = 81.5 - 0.05x$  dollars. The total cost for these blenders is given by  $C(x) = 0.06x^2 + 4.4x + 6000$  dollars. Determine the marginal profit for 144 blenders.

17. Find the indicated limit, if it exists.

$$\lim_{x \rightarrow 3} \left( \frac{x^2 - 9}{x^2 - 7x + 12} \right)$$

18. Consider the following function. Select the number of points of discontinuity for  $f(x)$ . Then enter each point and select its type of discontinuity.

$$f(x) = \frac{x-6}{x^2-4}$$

19. Use the Product Rule or Quotient Rule to find the derivative.

$$f(x) = (4x + 3)(4x^{-3} + 7)$$

20. It is estimated that  $t$  years from now the population of a city will number  $P(t) = (0.9t - 6)(0.4t + 5) + 10$  thousand people. How fast will the population (in thousands) be growing in 5 years? Round your answer to two decimal places.

21. Use the Product Rule or Quotient Rule to find the derivative.

$$f(x) = \frac{6x^4 + 5}{-2x^2 + 6}$$

22. The demand function for a particular item is  $D(x) = \frac{166}{2x+9}$ . Find the marginal revenue when  $x = 4$ . Round your answer to the nearest cent.

23. Find the derivative for the given function. Write your answer using positive and negative exponents and fractional exponents instead of radicals.

$$h(x) = (6x^2 + 4x + 9)^{\frac{3}{2}}$$

24. Evaluate  $\frac{dy}{dx}$  at  $x = 3$  for the function below.  
 $y = 4u^2 - u + 3$ , where  $u = 8x^2 + 5x + 7$

25. Find the derivative for the given function. Write your answer using positive and negative exponents and fractional exponents instead of radicals.

$$y = \sqrt[3]{5x^3 + 4}$$

26. It is estimated that  $t$  years from now the population of a city will be  $P(t) = 20(65 + 5t)^2 - 1100t$ .

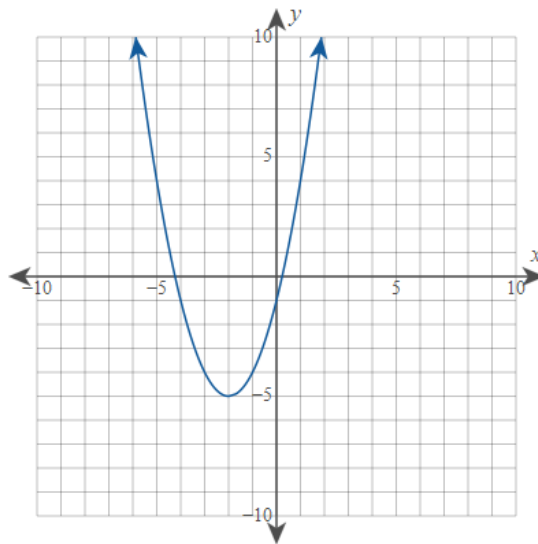
**Step 1.** What will the population be in 6 years?

**Step 2.** Find the rate of change in population in 9 years.

27. Determine the equation of the tangent line for  $f(x) = (2x^2 + 3x - 6)^2$  at the  $x$ -value given below.

$$x = -1$$

28. Consider the graph and determine the open intervals on which the function is increasing and on which the function is decreasing. Enter  $\emptyset$  to indicate the interval is empty.



29. Consider the function.

$$f(x) = -2x^3 - 12x^2 - 24x - 14$$

**Step 1.** Find all values of  $x$  that correspond to horizontal tangent lines. Select "None" if the function does not have any values of  $x$  that correspond to horizontal tangent lines.

**Step 2.** Determine the open intervals on which the function is increasing and on which the function is decreasing. Enter  $\emptyset$  to indicate the interval is empty.

30. A producer has determined that the revenue generated from the production and sale of  $x$  units of a product is given by  $R(x) = 215.568x - 0.009x^2$  dollars, where  $0 \leq x \leq 18100$ . For what interval(s) of production is the revenue increasing, and for what interval(s) is it decreasing? Write your answers as open intervals. Enter  $\emptyset$  to indicate the interval is empty.

31. Consider the function:

$$f(x) = 6(-3x^2 + 12)^2 + 6$$

**Step 1.** Find the critical values of the function.

**Step 2.** Use the First Derivative Test to find any local extrema. Enter any local extrema as an ordered pair.

32. The revenue from the sale of  $x$  manual food processors is given by  $R(x) = 29x - 0.2x^2$  dollars. The total cost is given by  $C(x) = 0.1x^2 - 10x + 780$  dollars, where  $0 \leq x \leq 100$ .

Determine the interval of sales for which the profit is increasing and the interval for which it is decreasing. Express your answer in open intervals.

33. Consider the function  $f(x) = 2x^3 - 24x$  on the interval  $[-8, 6]$ . Find the absolute extrema for the function on the given interval. Express your answer as an ordered pair  $(x, f(x))$ .

34. The weekly revenue from the production and sale of  $x$  units of coal is given by  $R(x) = 248x - 2x^2$  thousand dollars. The cost function is given by  $C(x) = x^2 + 200x + 3$  thousand dollars. Find the number of units of coal that are to be produced to maximize the profit if  $0 \leq x \leq 27$ .

35. The cost of producing  $x$  devices is given by  $C(x) = 95779 + 1.7x + 19x^2$  dollars. Find the value of  $x$  that minimizes the average cost function if  $1 \leq x \leq 137$ .

36. Consider the function:

$$f(x) = x^2 - 9\sqrt[4]{x} + 5 \text{ Find } f''(x).$$

37. Consider the function:

$$f(x) = -6x^3 - 108x^2 + 5x - 6$$

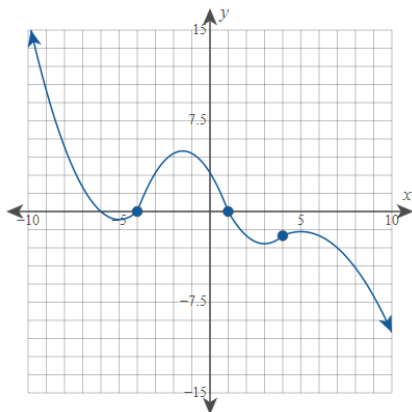
**Step 1.** Determine the intervals on which the function is concave upwards or concave downwards.

**Step 2.** Locate any points of inflection. Enter your answer as  $(x, y)$ -pairs.

38. The sales function for a product is given by  $S(x) = 133 + 19.8x^2 - 0.3x^3$ , where  $x$  represents **thousands** of dollars spent on advertising,  $0 \leq x \leq 44$ , and  $S$  is in **thousands** of dollars.

Find the point of diminishing returns. Enter the amount spent on advertising as well as the sales in dollars.

39. Consider the following graph.



**Step 1.** Determine the intervals on which the function is concave upward and concave downward.

**Step 2.** Determine the  $x$ -coordinates of any inflection point(s) in the graph.

40. Consider the function:

$$f(x) = -2x^3 + 12x^2 - 18x - 12$$

**Step 1.** Find the second derivative of the given function.

**Step 2.** Use the Second Derivative Test to locate any local maximum or minimum points in the graph of the given function.

41. Consider the function:

$$f(x) = (3x^2 + 10)^2$$

**Step 1.** Find the second derivative of the given function.

**Step 2.** Use the Second Derivative Test to locate any local maximum or minimum points in the graph of the given function.

42. Use the given conditions to draw a possible graph of the function  $f(x)$ .

a.  $f(2) = -1$

b.  $f'(2) = 0$

c.  $f'(x) < 0$  if  $x < 2$

d.  $f'(x) > 0$  if  $x > 2$

e.  $f''(x) > 0$  for all  $x$

43. Consider the following function:

$$f(x) = 2x^3 - 3x^2 - 36x + 5$$

**Step 1.** Determine  $f'(x)$  and  $f''(x)$ .

**Step 2.** Determine where the function is increasing and decreasing. Enter your answers in interval notation.

**Step 3.** Determine where the function is concave up and concave down. Enter your answers in interval notation.

**Step 4.** Enter the  $x$ -values of any local minima, maxima, and inflection points.

44. A bookstore sells 192 calculus textbooks per month at \$130 each. The owners estimate that for each \$13 increase in price, they will sell 6 fewer calculus textbooks per month. Find the price per calculus textbook that will maximize revenue.

45. Find the amount of money that will be accumulated in a savings account if \$4800 is invested at 7.0% for 14 years and the interest is compounded continuously. Round your answer to two decimal places.

46. If \$12,000 is invested at 9% compounded semi-annually, how much will this investment be worth in 20 years? Round your answer to two decimal places.

47. Helena's father is planning to open a savings account to pay for Helena's college education. He has found a bank that will pay 7 percent interest compounded semi-annually. How much will he need to deposit initially so that in 9 years the balance will be \$86,000? Round your answer to the nearest cent.

48. Use the properties of logarithms to expand the logarithmic expression as much as possible into the sum and/or difference of logarithmic expressions.  
**Note:** No term should contain exponents.

$$\ln(29x^6y)$$

49. Use the properties of logarithms to write the logarithmic expression as a single logarithm with no coefficients. Simplify.

$$5 \ln(2) - \ln(x) - \ln(17)$$

50. Use the properties of logarithms to write the logarithmic expression as a single logarithm with no coefficients. Simplify.

$$\ln(x^2 + 9x + 20) - \ln(x + 5)$$

51. Solve the following equation for  $x$ :

$$\ln(x - 4) + \ln(x - 1) = \ln(7)$$

52. How long does it take for \$2075 to double if it is invested at 7% compounded continuously? Round your answer to two decimal places.

53. The value of a machine,  $V$ , at the end of  $t$  years is given by  $V = C(1 - r)^t$ , where  $C$  is the original cost of the machine and  $r$  is the rate of depreciation. A machine that originally cost \$14,500 is now valued at \$6,496. How old is the machine if  $r = 0.09$ ? Round your answer to two decimal places.

54. The compound interest formula states that if  $P$  dollars are invested at an annual interest rate of  $r$ , compounded  $n$  times per year, then  $A$ , the amount of money present after  $t$  years, is given by  $A = P \left(1 + \frac{r}{n}\right)^{nt}$ . If \$10,000 is invested at 10% compounded quarterly, how long will it take for the balance to reach \$15,000? Round your answer to two decimal places.

55. Solve the following logarithmic equation using a calculator, if necessary, to evaluate the logarithm. Enter your answer as a fraction or round your answer to two decimal places.

$$\ln(4x + 6) = 4$$

56. Solve the following exponential equation for  $x$ . Write the answer as both an exact expression and as a decimal approximation (rounded to 2 decimal places).

$$2e^{4x-5} = 108$$

57. Consider the following function.

$$f(x) = 8x^2 + 6x - 9 + \ln(x)$$

**Step 1.** Determine a formula for  $f'(x)$ .

**Step 2.** Solve the equation  $f''(x) = 0$  for  $x$ . Write your answer in its simplest form or as a decimal rounded to the nearest thousandth.

**Step 3.** Find all possible inflection points in  $(x, f(x))$  form. Write your answer in its simplest form or as a decimal rounded to the nearest thousandth.

58. Find the absolute extrema for the given function on the interval  $[0.28, 5]$ . Write your answer in the form  $(x, f(x))$ . Round your answers to two decimal places.

$$f(x) = 3x^3 - 6 \ln(x^3)$$

59. A equipment dealer has estimated that he can sell  $N(x) = 340 + 37 \ln(4 + 3.5x)$  pieces of equipment annually, where  $x$  (in thousands of dollars) is the amount spent on advertising.

**Step 1.** Find the number of pieces of equipment sold if \$6000 is spent on advertising. Round your answer to the nearest integer.

**Step 2.** Find the rate of change in the number of pieces of equipment sold if \$6000 is spent on advertising. Round your answer to the nearest integer.

60. Find the derivative of the given expression.

$$f(x) = 16\sqrt{\ln(x)}$$

61. Consider the following function:

$$y = -4x^4 - 4e^x$$

**Step 1.** Find the first derivative of the above function.

**Step 2.** Find the second derivative of the above function.

62. Find the absolute extrema of the function over the given interval. Write your answers as ordered pairs. Round your final answer to four decimals.

$$g(x) = 9x^2e^{-0.7x}; [-1, 5]$$

63. The demand for a product is given by  $D(x) = 140e^{-0.05x}$ , where  $x$  is the number of units sold each week and  $0 \leq x \leq 85$ .

**Step 1.** Find the number of units sold that will yield maximum revenue. Round your answer to the nearest whole unit.

**Step 2.** Find the price per unit that will yield maximum revenue. Round your answer to the nearest cent.

64. Find the first derivative of the function.

$$f(x) = -3e^{\frac{1}{3}x^2 - 4}$$

65. Find the following indefinite integral.

$$\int (-10e^t + 7t) dt$$

66. Find the following indefinite integral.

$$\int \left( \frac{1}{y} - 7y^3 \right) dy$$

67. A manufacturer has determined that the marginal profit from the production and sale of  $x$  clock radios is approximately  $380 - 4x$  dollars per clock radio.

**Step 1.** Find the profit function if the profit from the production and sale of 38 clock radios is \$1700.

**Step 2.** What is the profit from the sale of 56 clock radios?

68. Find the antiderivative  $F(x)$  that satisfies the given condition.

$$\frac{dF}{dx} = 4x + 4e^x; F(0) = 17$$

69. Evaluate the definite integral below.

$$\int_1^5 (-5x^3 - 5) dx$$

70. Evaluate the definite integral below.

$$\int_1^2 (7 + 6e^{1.3x}) dx$$

71. The daily production level for a product is given by  $N(x) = 260 - 260e^{-0.2x}$  units, where  $x$  is the time in hours after production begins. Find the average production during the first 2 hours.

72. Find the total area bounded by the  $x$ -axis and the curve  $y = f(x)$  on the indicated interval. Enter your answer in exact form or as a decimal number rounded to the nearest thousandth.

$$f(x) = 3x^2 + 3x + 7; [-3, -1]$$

73. The marginal profit of a product is given by  $P'(x) = 23 - 0.042e^{0.2x}$  dollars per item, where  $x$  is the number of items produced and sold. Find the profit for the first 16 items. Round your answer to the nearest cent.

74. The marginal cost of a product is given by  $108 + \frac{53}{\sqrt{x}}$  dollars per unit, where  $x$  is the number of units produced. The current level of production is 55 units weekly. If the level of production is increased to 90 units weekly, find the increase in the total costs. Round your answer to the nearest cent.

75. Find the area of the region bounded by the graphs of the given equations.

$$y = x^2 + 16x + 1, y = -x^2 + 1$$

Math 105 - Final Exam Reviewn(Answer Key)

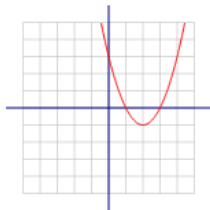
1. Correct Answer: 112
2. Correct Answer:  $\frac{1}{18}$
3. Correct Answer:  $\frac{1}{2}$
4. Correct Answer: 15
5. Step 1. Correct Answer: After 10 days 700 people have rented a certain movie.  
Step 2. Correct Answer: 3 is the rate of change in people who have rented a certain movie after 10 days.
6. Step 1. Correct Answer:  $f(5) = 5.415$   
The expected strikeouts per game was  $f(5)$  in 1982.  
  
Step 2. Correct Answer:  $f'(5) = 0.065$   
The rate of change in expected strikeouts per game was  $f'(5)$  in 1982.
7. Correct Answer:  $g'(x) = 7x^{-2}$
8. Correct Answer:  $f'(x) = -\frac{3}{2}x^{-\frac{3}{4}}$
9. Correct Answer:  $f'(x) = -8x + 6x^2$
10. Correct Answer:  $g'(x) = \frac{25}{2}x^{\frac{3}{2}} - \frac{9}{2}x^{\frac{1}{2}}$
11. Correct Answer:  $g'(x) = 48x^3 + 44x$
12. Step 1. Correct Answer:  
The slope of the tangent line at  $x = 1$  is  $-25$ .  
Step 2. Correct Answer:  $y = -25x + 12$
13. Step 1. Correct Answer: 23.32 barrels  
Step 2. Correct Answer: 3.98 barrels per hour  
Step 3. Correct Answer: 2.75 barrels per hour
14. Correct Answer:  $C(x) = 0.50x + 7300$
15. Correct Answer:  $\bar{C}'(x) = -\frac{1600}{x^2} + 0.3$
16. Correct Answer:  $P'(144) = 45.42$
17. Correct Answer:  $-6$
18. Correct Answer: 2, non-removable discontinuity at  $x = -2$ , non-removable discontinuity at  $x = 2$
19. Correct Answer:  $f'(x) = 28 - 32x^{-3} - 36x^{-4}$
20. Correct Answer: 5.7 thousand people per year
21. Correct Answer:  $f'(x) = \frac{-24x^5 + 144x^3 + 20x}{(-2x^2 + 6)^2}$
22. Correct Answer: \$5.17
23. Correct Answer:  $\frac{3}{2}(6x^2 + 4x + 9)^{\frac{1}{2}}(12x + 4)$
24. Correct Answer: 39803
25. Correct Answer:  $5x^2(5x^3 + 4)^{-\frac{2}{3}}$
26. Step 1. Correct Answer: 173900 people  
Step 2. Correct Answer: 20900 people/year
27. Correct Answer:  $y = 14x + 63$
28. Correct Answer: Increasing:  $(-2, \infty)$   
Decreasing:  $(-\infty, -2)$
29. Step 1. Correct Answer:  $-2$   
Step 2. Correct Answer: Increasing:  $\emptyset$ , Decreasing:  $(-\infty, \infty)$
30. Correct Answer: Increasing:  $(0, 11976)$ ,  
Decreasing:  $(11976, 18100)$
31. Step 1. Correct Answer:  $x = -2, 0, 2$   
Step 2. Correct Answer: Local Maxima:  $(0, 870)$ ,  
Local Minima:  $(-2, 6), (2, 6)$
32. Correct Answer: Increasing on  $(0, 65)$ , Decreasing on  $(65, 100)$
33. Correct Answer:  
Absolute Maximum:  $(6, 288)$   
Absolute Minimum:  $(-8, -832)$
34. Correct Answer:  $x = 8$  units
35. Correct Answer:  $x = 71$  devices
36. Correct Answer:  $f''(x) = 2 + \frac{27}{16}x^{-\frac{7}{4}}$
37. Step 1. Correct Answer: Concave Up:  $(-\infty, -6)$ ,  
Concave Down:  $(-6, \infty)$   
Step 2. Correct Answer: Points of Inflection:  
 $(-6, -2628)$
38. Correct Answer: Amount spent on Advertising: \$22000, Total Sales: \$6521800

39. Step 1. Correct Answer: Concave Up:  $(-\infty, -4), (1, 4)$ , Concave Down:  $(-4, 1), (4, \infty)$   
Step 2. Correct Answer:  $x = -4, 1, 4$

40. Step 1. Correct Answer:  $f''(x) = -12x + 24$   
Step 2. Correct Answer: Local Maxima:  $(3, -12)$ ,  
Local Minima:  $(1, -20)$

41. Step 1. Correct Answer:  $f''(x) = 108x^2 + 120$   
Step 2. Correct Answer: Local Maxima: No Local  
Maxima, Local Minima:  $(0, 100)$

42. Correct Answer:



43. Step 1. Correct Answer:  $f'(x) = 6x^2 - 6x - 36$ ,  
 $f''(x) = 12x - 6$   
Step 2. Correct Answer: Increasing on  
 $(-\infty, -2), (3, \infty)$ , Decreasing on  $(-2, 3)$   
Step 3. Correct Answer: Concave up on  $(\frac{1}{2}, \infty)$ ,  
Concave down on  $(-\infty, \frac{1}{2})$   
Step 4. Correct Answer: Local minimum at  $x = 3$ ,  
Local maximum at  $x = -2$ , Inflection pt at  $x = \frac{1}{2}$

44. Correct Answer: \$273 per calculus textbook.

45. Correct Answer: \$12789.39

46. Correct Answer: \$69796.37

47. Correct Answer: \$46299.06

48. Correct Answer:  $\ln(29) + 6 \ln(x) + \ln(y)$

49. Correct Answer:  $\ln(\frac{32}{17x})$

50. Correct Answer:  $\ln(x + 4)$

51. Correct Answer:  $\frac{5 + \sqrt{37}}{2}$

52. Correct Answer: 9.90 years

53. Correct Answer: 8.51 years

54. Correct Answer: 4.11 years

55. Correct Answer:  $x = 12.15$

56. Correct Answer:  $\frac{1}{4} \ln(54) + \frac{5}{4} \approx 2.25$

57. Step 1. Correct Answer:  $f''(x) = 16 - \frac{1}{x^2}$   
Step 2. Correct Answer:  $x = \frac{1}{4}$   
Step 3. Correct Answer:  $(\frac{1}{4}, -7 + \ln(\frac{1}{4}))$

58. Correct Answer: Absolute Minimum:  $(1.26, 1.84)$ ;  
Absolute Maximum:  $(5, 346.03)$

59. Step 1. Correct Answer:  $N(6) = 459$   
Step 2. Correct Answer: 5

60. Correct Answer:  $\frac{8}{x\sqrt{\ln(x)}}$

61. Step 1. Correct Answer:  $-16x^3 - 4e^x$   
Step 2. Correct Answer:  $-48x^2 - 4e^x$

62. Correct Answer:  $(0, 0), (-1, 18.1238)$

63. Step 1. Correct Answer: 20 units  
Step 2. Correct Answer: \$51.50

64. Correct Answer:  $-2xe^{\frac{1}{3}x^2 - 4}$

65. Correct Answer:  $-10e^t + \frac{7}{2}t^2 + C$

66. Correct Answer:  $\ln(|y|) - \frac{7}{4}y^4 + C$

67. Step 1. Correct Answer:  $P(x) = 380x - 2x^2 - 9852$   
Step 2. Correct Answer: \$5156

68. Correct Answer:  $F(x) = 2x^2 + 4e^x + 13$

69. Correct Answer: -800

70. Correct Answer:  $7 + \frac{60}{13}(e^{2.6} - e^{1.3}) \approx 52.21$

71. Correct Answer:  $260 + 650(e^{-0.4} - 1) \approx 46$   
units

72. Correct Answer: 28

73. Correct Answer: \$363.06

74. Correct Answer: \$3999.49

75. Correct Answer:  $\frac{512}{3}$