MATH 111 CHAPTER 2 (sec. 2.1-2.10)

1. Terms to know: function, the domain and range of the function, vertical line test, even and odd functions, rational power function, vertical and horizontal shifts of a function, reflection in the x-axis, reflection in the y-axis, vertical stretching and compressions of a function, constant function, linear function, point-slope equation of a line, slope-intercept equation of a line, slope of a line, parallel and perpendicular lines, quadratic function, axis of symmetry, vertex, minimum/maximum value, increasing/decreasing, standard (vertex) form of a quadratic function, piece-wise defined function, continuous and discontinuous function, absolute value function, arithmetic combinations of functions, function composition, inverse function, one-to-one function, horizontal line test, difference quotient, properties of inverse functions, optimization problem.
2. Be able to evaluate functions and simplify their result. (sec. 2.1)
3. Be able to determine domain and range of the function. (sec. 2.1)
4. Know and be able to use vertical and horizontal shifts, vertical stretching and compressions, and reflections to graph functions. (sec. 2.2)
5. Know the basic shapes, domain and range, and sketch the graph of other common functions in the form y = $x^{m/n}$. (sec. 2.2)
6. Be able to determine if a given function is even, odd or neither even nor odd. (sec. 2.2)
7. Be able to find the slope-intercept equation, and point-slope equation of a line. (sec. 2.3)
8. Be able to find the equation of the line that passes through the point and parallel or (and) perpendicular to the given line. (sec. 2.3)
9. Know the applications of the linear functions. (sec. 2.3)
10. Be able to use the completion of squares technique to rewrite quadratic function in the vertex form $f(x) = a(x-h)^2 + k$.(sec. 2.4)
11. Be able to find the x- and y-intercepts, axis of symmetry, vertex, minimum/maximum value, increasing/decreasing intervals, domain and range and sketch the graph of the quadratic function. (sec. 2.4)
12. Be able to use the quadratic formula to factor and solve the quadratic equation. (sec. 2.4)
13. Know the applications of quadratic functions. (sec. 2.4)
14. Be able to determine a quadratic equation from the given graph of a quadratic function. (sec. 2.4)
15. Be able to evaluate, graph, and find the domain and range of a piece-wise defined and an absolute value functions. (sec. 2.5)
16. Know the arithmetic combinations of functions, be able to find them, and give the domain for each combination. (sec. 2.6)
17. Be able to find composition of two and three functions and give the domain for each composition. (sec. 2.6)
18. Be able to write the function as the composition of few functions. (sec. 2.6)
19. Be able to find an inverse function. (sec. 2.8)
20. Be able to find the domain and range of an inverse function. (sec. 2.8)
21. Be able to write an equation of the function in terms of one variable based on the given information. (sec. 2.9)
22. Be able to build a function from words. (sec. 2.9)
23. Be able to find and simplify difference quotient $\frac{f(x+h)-f(x)}{h}$.(sec. 2.10)
24. Any handout given in class, any class discussions.

Partial Review Exercises

1. Write the function in the form $f(x) = a(x-h)^2 + k$: a). $f(x) = 2x^2 - 12x + 11$; b) $y = -\frac{1}{2}x^2 + x + \frac{5}{2}$;

c) $f(x) = -x^2 + 8x - 7$. SLO (Student Learning Outcome) 10

2. Given $f(x) = \sqrt{x}$ and $g(x) = \sqrt[3]{5-x}$. Find a). $(f \circ g)(x)$; b). $(g \circ f)(x)$ and give the domain for each composition. SLO 17

3. Solve the given equation: a) $x^2 + 8x - 1 = 0$; b) $9 + 13x - 10x^2 = 0$. SLO 12

4. If a projectile is shot vertically upward from the ground with an initial velocity of 100 feet per second, neglecting air resistance, its height s (in feet) above the ground t seconds after projection can be modeled by $s = -16t^2 + 100t$. SLO 13

a) How long will it take for the projectile to return to the ground?

b) When will it reach maximum height?

c) What is the maximum height?

5. Sketch the graph SLO 15

a). $f(x) = \begin{cases} \sqrt{x}, x \geq 0 \\ -2, x < -1 \end{cases}$; b). $f(x) = \begin{cases} x^2, x < 1 \\ x + 1, x > 1 \end{cases}$; c) $y = |x^2 + x - 6|$; d) $f(x) = \begin{cases} x + 4, x < 0 \\ 1 - x, x \geq 0 \end{cases}$;

e) $f(x) = \begin{cases} 3 + x, x < -1 \\ x^2, -1 \leq x < 1 \\ 2 - x, x \geq 1 \end{cases}$.

6. An open rectangular box with a volume of 12 ft^3 has a square base. Express the surface area A of the box as a function of the length x of the side of the base. SLO 21, 22

7. Find the number of units that produce the maximum revenue, $R = 900x - 0.1x^2$, where R is the total revenue in dollars and x is the number of units sold. SLO 11

8. Given $f(x) = \sqrt{x-7}, g(x) = x^2, h(x) = x^3 + 6$. Find $f \circ g \circ h$. SLO 17

9. Given f(x)= $\frac{3}{x+1}$; g(x)= $\sqrt{2-x}$. SLO 17

Find a). f ∘ g; b). g ∘ f and give the domain for each composition.

10. Evaluate (if possible) the function $f(x) = \frac{x}{x-2}$. Simplify your answer. SLO 2

a). f(2t); b). f(s+2); c). f(1/x); d). 1/f(x); e).f $\sqrt{x+2}$; f). f(x+h) – f(x).

11. Given f(x) = $\sqrt{x}$ and g(x) = $\sqrt{4-x^2}$. SLO 16

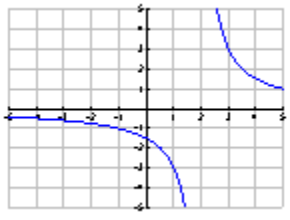
Find a). $(\frac{f}{g})$ (x); b). (fg)(x) and give the domain for each combination.

12. Find $\frac{f(x+h) - f(x)}{h}$ for the following functions: a). $f(x) = 7x - 3$; b). $f(x) = x^2 + 2x$;

c). $f(x) = \frac{2}{x-5}$; d). $f(x) = \sqrt{x+3}$. SLO 23

13. Use the graphs to determine the domain and range of the given function SLO 3, 5

a). b).



14. Determine the domain for the given function SLO 3

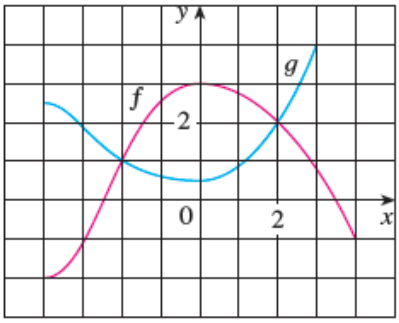
a). $f(x) = |x-3|$ b). $f(x) = \dfrac{2}{x+5}$ c). $f(x) = \dfrac{1}{\sqrt{2x+1}}$

d). $f(x) = \sqrt{\dfrac{(x+2)^2}{x^2-5x+6}}$ e). $g(x) = \sqrt{x(2-x)}$ f). $g(x) = \dfrac{x^4-x^2}{x^2-1}$.

15. A farmer wants to enclose a rectangular field by a fence and divide it into two smaller rectangular fields by constructing another fence parallel to one side of the field. He has 3000 yards of fencing. Find the dimensions of the field so that the total area is a maximum. SLO 11, 21, 22

16. Express the area A of a circle as a function of its circumference C. SLO 21, 22

17. The graphs of f and g is given SLO 2, 3



a) State the values of $f(-3)$ and $g(-2)$.
b) Estimate the solutions of the equation $f(x) = -1$.
c) On what interval is f decreasing?
d) State the domain and range of f.
e) State the domain and range of g.

18. An open box is to be made from a square piece of material 32 centimeters on a side by cutting equal squares (x centimeters) from the corners and turning up the sides. Determine the volume of the box as the function of x. SLO 21, 22

19. Find the inverse function SLO 19

a). $f(x) = \dfrac{2x+5}{x-1}$; b). f(x) = 2x – 5; c). $y = 2 + \sqrt{x-3}$; d). $y = (x-2)^2$, x ≥ 2; e) $f(x) = \dfrac{x}{x+3}$.

20. A **closed** rectangular box with a square base x is and height h to be constructed from 300 square inches of material. Express the volume of the box as a function of the length x of the side of the base. SLO 21, 22

21. Use the table below to find the following values SLO 16, 17

x	f(x)	g(x)
0	4	4
1	2	3
2	0	2
3	1	1
4	4	0

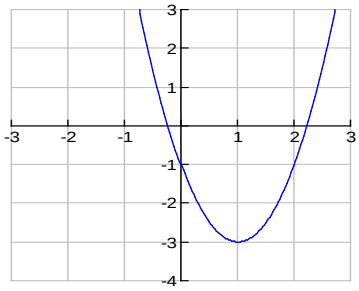
Find a). (f ∘ g)(2); b). (g ∘ f)(2); c) (f + g) (3).

22. Find the maximum or the minimum value of $f(x) = -2x^2 - 10x + 5$. SLO 11

23. Find the largest interval on which $f(x) = -(x+1)^2$ is increasing. SLO 11

24. Express the distance d, from a point (x, y) on the graph of $x + y = 3$ to the point (5, 8) as a function of x. SLO 21, 22

25. Determine the equation of the quadratic function whose graph is shown below. SLO 14



26. Determine whether each of the given functions is even, odd, or neither even nor odd. SLO 6

a). $f(x) = \dfrac{x^2 - 1}{x^3}$ b). $f(x) = x^6 - 4$.

27. Find the equation of the line passing through (-1, 3) and perpendicular to the line 2x + 3y = 9. SLO 8

28. Fund functions f and g such that $F(x) = f \circ g$. SLO 18

a) $F(x) = (x+1)^2 - \frac{4}{x+1}$; b) $F(x) = \sqrt[3]{5x-1}$.

29. As dry air moves upward, it expands and cools. If the ground temperature is 20^0C and the temperature at a height of 1 km is 10^0C, express the temperature T in terms of the height h. SLO 9

30. From ABC Wireless the monthly cost for a cell phone with 100 minutes per month is $35, or 200 minutes per month is $50. SLO 9

a) Given that the cost in dollars is a linear function of the time in minutes, find a formula for the cost function.

b) What is the cost of 400 minutes per month?

31. Sketch the graph of a) $f(x) = -(x-3)^{3/2} + 1$; b) $f(x) = (3-x)^{2/3} - 1$; c) $f(x) = \sqrt{x+2} + 1$. SLO 4, 5

32. Find an equation of the line passing through the points (4,3) and (-2,5). SLO 7

33. Given $f(x) = \frac{1-x}{x-2}$. Find $f^{-1}(x)$ and state the domain and range of $f^{-1}(x)$. SLO 20

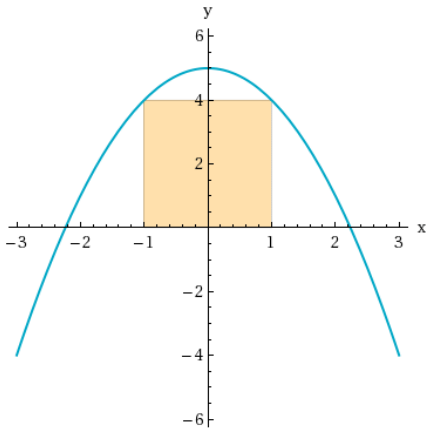
34. If $f(x) = 2x^2 - x + 2$, find the following SLO 2

a) $f(a+1)$; b) $f(a^2)$; c) $[f(a)]^2$; d) $f(a+h)$.

35. Evaluate $\dfrac{f(3+h) - f(3)}{h}$ for the function $f(x) = 1 + 2x - x^2$. SLO 23

36. Evaluate $\frac{f(a+h)-f(a)}{h}$ for the function $f(x)=-x^3$. SLO 23

37. A rectangle is inscribed in the parabola $y=-x^2+5$ as shown, with its base on the x-axis. SLO 21, 22



Write the area A of the rectangle as a function of x.

Answers:

1. a). $2(x-3)^2-7$; b). $-\frac{1}{2}(x-1)^2+3$; c) $-(x-4)^2+9$.

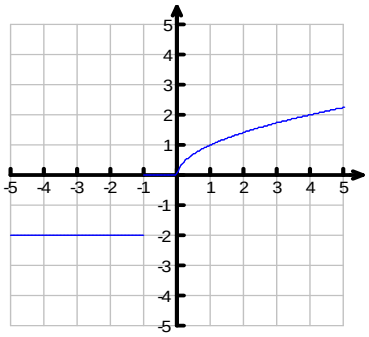
2. a) $\sqrt[6]{5-x}$, D $(-\infty,5]$; b) $\sqrt[3]{5-\sqrt{x}}$, D$[0,\infty)$.

3. a) $x=-4\pm\sqrt{17}$; b) $x=-1/2; x=9/5$.

4. a). 6.25 s., b) 25/8 s; c) 625/4 ft.

5.

a)



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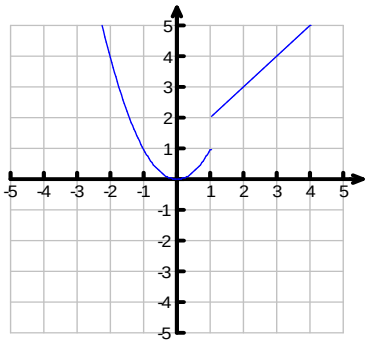

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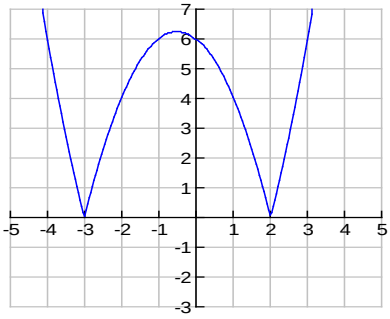

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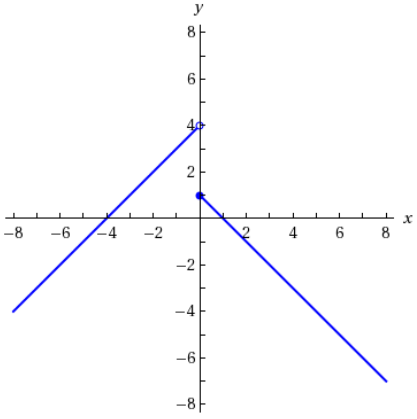
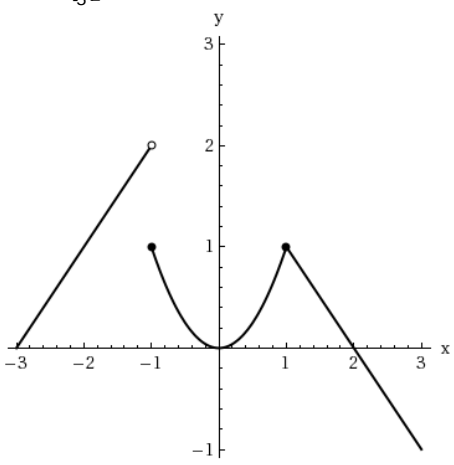


e)



6. $A = x^2 + \frac{48}{x},\ x > 0$.

7. 4500 units

8. $\sqrt{(x^3+6)^2 - 7}$.

9. a). $\frac{3}{\sqrt{2-x}+1}$; D: $(-\infty, 2]$; b). $\sqrt{\frac{2x-1}{x+1}}$; D: $(-\infty,-1) \cup [1/2, \infty)$;

10. (a) $\frac{t}{t-1}$;(b). $\frac{s+2}{s}$;(c). $\frac{1}{1-2x}, x \neq 0$;(d) $\frac{x-2}{x}, x \neq 2$;(e) $\frac{\sqrt{x+2}}{\sqrt{x+2}-2}$; (f) $\frac{-2h}{(x+h-2)(x-2)}$.

11. a). $\sqrt{\frac{x}{4-x^2}}$; D:$[0, 2)$. b). $\sqrt{4x - x^3}$; D: [0, 2].

12. (a). 7 ; (b). 2x + h +2; (c). $\frac{-2}{(x+h-5)(x-5)}$; (d). $\frac{1}{\sqrt{x+h+3}+\sqrt{x+3}}$.

13. (a) domain: [-2, $+\infty$), range: [1, $+\infty$);(b). $(-\infty, 2) \cup (2, \infty)$, range $(-\infty, 0) \cup (0, \infty)$.

14. (a). $(-\infty, +\infty)$; (b). all x's except x = -5; (c). (-1/2, $+\infty$); (d). $(-\infty, 2) \cup (3, \infty)$; (e). [0,2]; f). $(-\infty,-1) \cup (-1,1) \cup (1, \infty)$.

15. 500 by 750

16. A = $\frac{C^2}{4\pi}$.

17. a) $f(-3) = -1; g(-2) = 1$; b) $x = -3, x = 4.$; c) (0,4); d) Domain:[-4, 4], Range: [-2, 3]; e) Domain: [-4, 3], Range [.5, 4].

18. V(x) = $x(32 - 2x)^2$;

19. a). $f^{-1}(x) = \frac{5+x}{x-2}$; b). $\frac{x+5}{2}$; c). $(x-2)^2 + 3$, x ≥ 2; d). $\sqrt{x} + 2$; e) $f^{-1}(x) = \frac{3x}{1-x}$.

20. $V = 75x - \frac{x^3}{2}$.

21. a). 0, b). 4, c). 2.

22. 35/2.

23. $(-\infty, -1]$.

24. $d = \sqrt{2x^2 + 50}$.

25. $y = 2(x-1)^2 - 3$.

26. (a). odd (b). even.

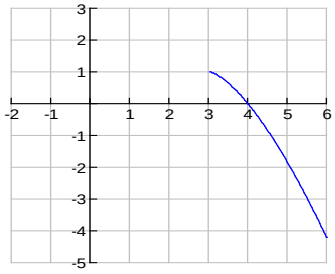
27. y = 3/2x + 9/2.

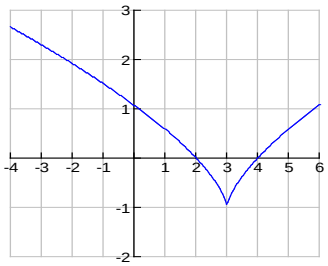
28. a) $f = x^2 - \frac{4}{x}$; $g = x + 1$; b) $f = \sqrt[3]{x}; g = 5x - 1$.

29. T = -10h + 20.

30. a) y = .15x + 20
b) $80

31. a) b) c)





32. y = -1/3 x + 13/3.

33. $f^{-1}(x)=\frac{2x+1}{x+1}$; domain of $f^{-1}(x):(-\infty,-1)\cup(-1,\infty)$; range of $f^{-1}(x):(-\infty,2)\cup(2,\infty)$.

34. a) $2a^2+3a+3$; b) $2a^4-a^2+2$; c) $4a^4-4a^3+9a^2-4a+4$; d) $2a^2+4ah+2h^2-a-h+2$.

35. $-h-4$.

36. $-3a^2-3ah-h^2$.

37. $A(x)=10x-2x^3$.