MATH 111 CHAPTER 4 Review (sections 4.1-4.5 + 4.7-4.8 + 4.10-4.13)

1. Terms to know: radian and degree measure, reference angle, arc length, area of a circular sector, Pythagorean identities, periodic functions, unit circle, amplitude, period, phase shift, sum and difference formulas, double-angle formulas, trigonometric equations, inverse trigonometric functions: arcsine, arccosine, arctangent, opposite, adjacent, hypotenuse sides, law of sines, law of cosines.
2. Be able to convert from degree measure to radian measure and from radian measure to degree measure. (sec 4.1)
3. Be able to calculate the arc length subtended by a central angle and the area of a circular sector. (sec 4.1)
4. Know and be able to find the values for all six trig functions: sine, cosine, tangent, cotangent, secant, cosecant at $\pi$ n/2, $\pi$ n/6, $\pi$ n/4, $\pi$ n/3. (sec 4.2, 4.4)
5. Know the unit circle. (sec 4.2)
6. Know and be able to use Pythagorean identities. (sec 4.2, 4.5)
7. Know the domain and range of all six trig functions. (sec 4.3, 4.4 )
8. Be able to find period, phase shift, and graph sine, cosine, tangent, cotangent, secant, and cosecant functions with any given shifts, stretches/shrinks. (sec 4.3, 4.4 )
9. Be able to find an equation of the sine or cosine curve from the given graph. (sec 4.3)
10. Be able to find an equations all asymptotes and x-intercepts (if any) of the sine, cosine, tangent, cotangent, secant, cosecant curves. (sec 4.3, 4.4 )
11. Be able to use: the sum and difference formulas and double- angle formulas for the sine, cosine and tangent. (sec 4.5)
12. Be able to find the values of tan t, cot t, sec t, csc t in terms of sin t and/or cos t . (sec 4.4)
13. Be able to solve trig equations algebraically and graphically. (sec 4.2, 4.8 )
14. Be able to solve trig equations: (1) simplify using trig identities, (2) solve. (sec 4.2, 4.5, 4.8 )
15. Know the domain, range, and graphs of the inverse trig functions: arcsin x, arcos x, arctan x. (sec 4.7 )
16. Be able to find a value of the inverse function without a calculator (using the definition. (sec 4.7 )
17. Know the properties of inverse trigonometric functions. (sec 4.7 )
18. Know the trig functions of an angle in a right triangle (be able to solve for unknown). (sec 4.10)
19. Know applications of Right Triangle. (sec 4.11)
20. Know Law of Sines and Law of Cosines and be able to use them to solve oblique triangle problems. (sec 4.12, 4.13)
21. Any handouts given in class, any class discussions.

**Review Exercises:**

1. Solve the equations on $[0,2\pi]$, SLO 13, 14

a) $2\sin^2 x+\sin x-1=0$, b) $\cot x+\sqrt{3}=0$ on $(0,2\pi)$ .

c) $3\sin x=2\cos^2 x$, d) sin x = cos x ; e) $2\sin^2 x-\sin x-1=0$;

f) $2\sin^2 x+3\cos x-3=0$; g) $3\tan^2 x-1=0$; h) 2cos(3t)-1 = 0; i) ) sin x = $-\frac{\sqrt{3}}{2}$.

2. Find the amplitude, period, Horizontal shift and sketch one period of each of the given SLO 8, 10

a). $y = 5\cos(\frac{6}{5}x)$; b). f(x) = $6\sin(3x - \frac{\pi}{4})$ c). f(x) = tan (x + $\frac{\pi}{4}$); d) $y = -5\cot(2x)$; e) $y = -3\csc(\frac{x}{4})$.

3. Find the exact value of

a). sin $\left(\frac{7\pi}{6} - \frac{3\pi}{4}\right)$ b). cos $\frac{\pi}{12}$ c). $\cos(\frac{\pi}{6} + \frac{\pi}{4})$. SLO 11

4. Find the following: a). $\sin(-\frac{3\pi}{4})$; b). sin $(7\pi/3)$; c). $\cos(\frac{5\pi}{6})$ d). cos $(25\pi/6)$; e).$\sin(\frac{3\pi}{2})$;

f). cos(-7 $\pi$/3); g) $\sin(240^0)$; h) $\cos(240^0)$. SLO 4, 5

5. Convert from degrees to radians: a) $165^0$; b) $-150^0$. SLO 2

6. Evaluate

a) $\sec(\frac{5\pi}{6})$, b) $\tan(\frac{2\pi}{3})$, c) $\cot(\frac{\pi}{6})$, d) $\csc(\frac{\pi}{4})$. SLO 4

7. Find the exact value of each of the given: SLO16, 17

a). sin(arctan $\frac{3}{4}$); b). cos(arcsin 5/13); c). $\cos^{-1}(-\frac{1}{2})$; d). $\sin^{-1}(\frac{1}{2})$; e). $\sin(\sin^{-1}\frac{2}{3})$;

f) $\tan^{-1}(-\frac{\sqrt{3}}{3})$; g) arcsin(-2); h) $\cos(\arctan\frac{12}{5} + \arcsin\frac{3}{5})$; i) $\sin(arc\cos\frac{1}{3} - \arcsin\frac{2}{5})$,

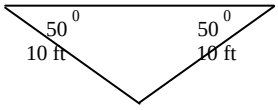
j) $\arcsin(\sin\frac{5\pi}{3})$; k) $\arccos(\pi)$; l) cos(arccos -.2); m) sin(arcsin 3); n) $\sin^{-1}(\sin(\frac{5\pi}{7}))$.

8. If $\sec x = \frac{3}{2}$ and $\tan x < 0$, find the value of $\sin(2x)$. SLO 11

9. If sin a = 3/5, $0 < a < \frac{\pi}{2}$ and cos b = $\frac{2}{\sqrt{5}}, -\frac{\pi}{2} < b < 0$, find

a). sin (a +b) b). sin 2a. SLO 11

10. In preparation for an outdoor rock concert, a stage crew must determine how far apart to place the two large speaker columns on stage. What generally works best is to place them at $50^0$ angles to the center of the front row. The distance from the center of the front row to each of the speakers is 10 ft. How far apart does the crew need to place the speakers on stage? Round your answer to one decimal place. SLO20




11. The angles of elevation to an airplane from two points A and B on level ground are $51^0$ and $68^0$, respectively. The points A and B are 6 mi apart, and the airplane is between these positions in the same vertical plane. Find the altitude of the airplane. SLO20, 21

12. If the sun is $30^0$ above the horizon, find the length of a shadow cast by a silo that is 70 feet high. SLO19

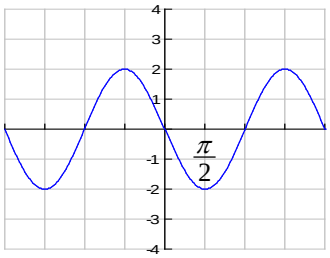
13. Find all solutions of the given equation:

a) $\sin(2x)+2\sin x-2\cos x=2$; b) $4\sin^2 x-8\sin x+3=0$; c) $2\cos^2 x+\cos x-1=0$;

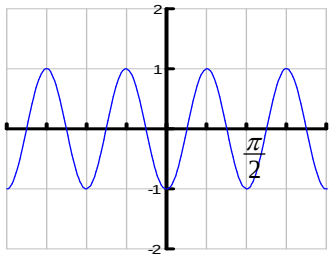
d) $\tan^2(2x)=1$. SLO 14

14. Find an equation of the cosine function whose graph matches the given curve SLO 9

a)




b)




15. If $\tan x=-2$ and $\frac{\pi}{2}<x<\pi$, find the value of $\cos x$. SLO 6, 12, 19

16. Find the arc length subtended by a central angle of $30^0$ in a circle of radius 2 cm. SLO 3

17. Find both the degree and the radian measures of the smallest positive angle formed by the hands of a clock at 3:30. SLO 2

18. If $\sin t=\frac{3}{8}$ and $\frac{\pi}{2}\le t\le\pi$, find the value of cos t. SLO 6

19. Convert from radians to degrees: a) $\frac{4\pi}{3}$; b) $\frac{7\pi}{12}$. SLO 2

20. A 16-in pizza is cut into 8 slices. Find the area of one slice. SLO 3

21. If $\csc x=-5, \frac{3\pi}{2}\le x\le 2\pi$, determine the values of a) $\tan x$ and b) $\cos(2x)$. SLO 11, 12

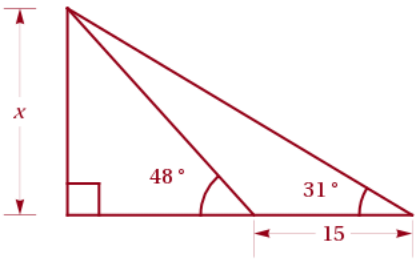
22. Write $\cos(\tan^{-1} x)$ as an algebraic expression in $x$. SLO 16, 17

23. Find the solutions of the given equation in the indicated interval

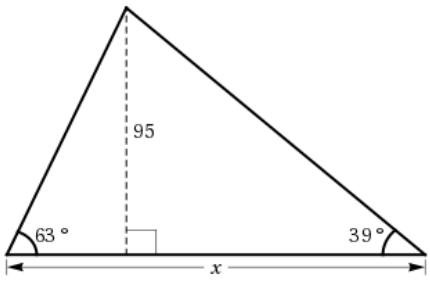
a) $4\cos^2 x-3\cos x-2=0$ on $[0,\pi]$; b) $\tan^2 x+\tan x-1=0$ on $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. SLO 13, 17

24. Solve for x. SLO 18

a)




b)




25. A tracking telescope, located 1.59 km from the point of a rocket launch, follows a vertically ascending rocket. Express the height *h* of the rocket in kilometers as a function of the angle of elevation $\theta$. SLO 19

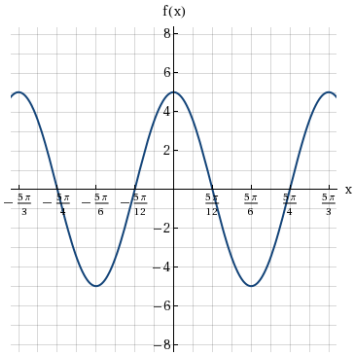
26. Two points *A* and *B* lie on opposite sides of a river. Another point *C* is located on the same side of the river as *B* at a distance of 220 ft from *B*. If angle *ABC* is 105° and angle *ACB* is 25°, find the distance across the river from *A* to *B*. (Round your answer to two decimal places.) SLO 20

27. Buoys *A*, *B*, and *C* mark the vertices of a triangular racing course on a lake. Buoys *A* and *B* are 450 meters apart, buoys *B* and *C* are 650 meters apart, and buoys *A* and *C* are 550 meters apart. Find the smallest angle of this race course to the nearest tenth degree. SLO 20

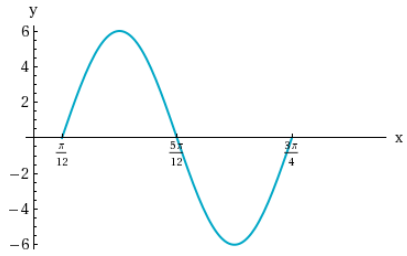
Answers:

1. a) $x = \pi/6, 5\pi/6, 3\pi/2$ ; b) $x = \frac{5\pi}{6}; \frac{11\pi}{6}$, c) $x = \frac{\pi}{6}, \frac{5\pi}{6}$; d) $x = \frac{\pi}{4}, \frac{5\pi}{4}$; e) $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$;
   f) $0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$ ; g) $\frac{\pi}{6}, \frac{5\pi}{6,}, \frac{7\pi}{6}, \frac{11\pi}{6}$ ; h) $\frac{\pi}{9}, \frac{5\pi}{9}\frac{7\pi}{9}\frac{11\pi}{9}\frac{13\pi}{9}\frac{17\pi}{9,}$ i) x = $4\pi/3$, $5\pi/3$.

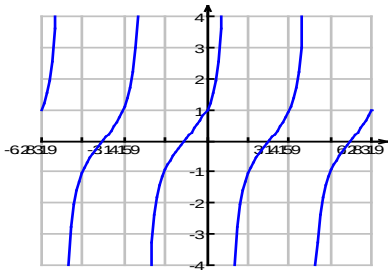
2. a). ampl. = 5
   period = $\frac{5\pi}{3}$
   hor. shift = none



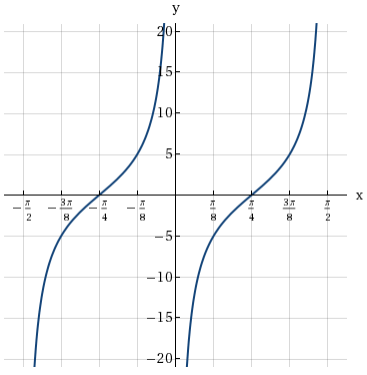

   b). ampl. = 6
   period = $2\pi/3$
   hor. shift = $\frac{\pi}{12}$




   c). ampl. = none
   period = $\pi$ ; VA $x = \frac{\pi}{4} + \pi n$
   hor. shift = $-\frac{\pi}{4}$

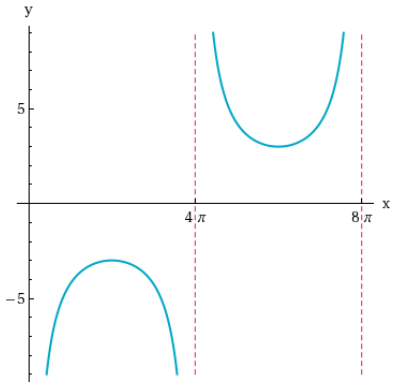


   d)




   Period = $\frac{\pi}{2}$;
   VA: $x = \frac{\pi}{2}n$.

   e)




   Period = $8\pi$;
   VA: $x = 4\pi n$.

3. a). $\frac{1}{4}(\sqrt{2}+\sqrt{6})$; b). $\frac{1}{4}(\sqrt{2}+\sqrt{6})$; c). $\frac{1}{4}(\sqrt{6}-\sqrt{2})$.

4. . a). $-\frac{\sqrt{2}}{2}$; b). $\frac{\sqrt{3}}{2}$; c). $-\frac{\sqrt{3}}{2}$; d). $\frac{\sqrt{3}}{2}$; e).-1; f).1/2, g) $-\frac{\sqrt{3}}{2}$; h) -1/2.

5. a) $\frac{11\pi}{12}$; b) $-\frac{5\pi}{6}$.

6. a) $\frac{-2\sqrt{3}}{3}$, b) $-\sqrt{3}$, c) $\sqrt{3}$, d) $\sqrt{2}$.

7. a). 3/5; b). 12/13; c). $\frac{2\pi}{3}$; d). $\frac{\pi}{6}$; e). 2/3; f) $-\frac{\pi}{6}$; g) dne; h) $-\frac{16}{65}$; i) $\frac{2(\sqrt{42}-1)}{15}$, j) $-\frac{\pi}{3}$; k) dne; l) -.2; m) dne; n) $\frac{2\pi}{7}$.

8. $-\frac{4\sqrt{5}}{9}$

9. a). $\frac{2\sqrt{5}}{25}$, b). $\frac{24}{25}$.

10. 12.9 ft

11. 4.94.

12. 121. 2.

13. a) $\frac{\pi}{2}+2\pi n;\ \pi+2\pi n$; b) $\frac{\pi}{6}+2\pi n;\ \frac{5\pi}{6}+2\pi n$; c) $\frac{\pi}{3}+2\pi n, \frac{5\pi}{3}+2\pi n, \pi+2\pi n$; d) $\frac{\pi}{8}+\frac{\pi}{4}n$.

14. a) y = 2cos(x + $\frac{\pi}{2}$); b). y = - cos(4x )

15. $-\frac{1}{\sqrt{5}}$.

16. $\frac{\pi}{3}$.

17. $\frac{5\pi}{12}, 75^0$.

18. $-\sqrt{55}/8$.

19. a) $240^0$; b) $105^0$.

20. $8\pi$.

21. a) $-\frac{\sqrt{6}}{12}$; b) $\frac{23}{25}$.

22. $\frac{1}{\sqrt{x^2+1}}$.

23. a) $\cos^{-1}\left(\frac{3-\sqrt{41}}{8}\right)$; b) $\tan^{-1}\left(\frac{-1-\sqrt{5}}{2}\right), \tan^{-1}\left(\frac{-1+\sqrt{5}}{2}\right)$.

24. a) 19.64; b) 165.72.

25. $1.59\tan\theta$.

26. 121.37 ft.

27. $43^0$.