Math 101 Review on Polynomials & Rational Functions

Learning Goals

☐ (PY-1) Properties of Polynomial functions
I can identify polynomial functions and their degree. I can graph polynomial functions using transformations. I can analyze the graph of polynomial functions. I can identify the real zeros of polynomial functions and their multiplicity.

☐ (PY-2) Properties of Rational functions
I can find the domain of a rational function. I can find the vertical asymptotes of a rational function. I can find the horizontal asymptotes of a rational function. I can find the oblique asymptotes of a rational function. I can find any intercepts appearing on the graph of a rational function.

☐ (PY-3) Polynomial and Rational inequalities
I can produce a completely correct Sign Chart without any errors. I can solve polynomial inequalities (including quadratic). I can solve rational inequalities.

Review Problems

1. For each polynomial function, determine the degree, the leading coefficient, and each real zero with its multiplicity.

   (a) $f(x) = x^2 - 4x + 3$ (b) $f(x) = (2x + 10)^2$ (c) $f(x) = (x - 2)^5$

   (d) $f(x) = x^3 - 8$ (e) $f(x) = 4(x + 4)(x + 3)^3$ (f) $f(x) = -x^2(x^2 - 25)$

2. For each polynomial, determine which of the following descriptions apply:

   I. The graph rises on the left.
   II. The graph falls on the left.
   III. The graph rises on the right.
   IV. The graph falls on the right.

   (a) $f(x) = -x^5 - 7x^4 + 6x^3 - 9$ (b) $f(x) = -4x^4 + 6x^3 - 2x - 5$

   (c) $f(x) = 2x(x - 1)^2(x + 3)$ (d) $f(x) = -x(x + 3)^2(x - 3)^2$

3. For each rational function below, find the domain, find the equations of any asymptotes (vertical, horizontal, or oblique), and state the location of any intercepts appearing on each graph.

   (a) $f(x) = \dfrac{3x}{x^2 + 2}$ (c) $f(x) = \dfrac{x^2 - x - 6}{x^2 + x - 6}$ (e) $f(x) = \dfrac{(x + 1)(x + 3)}{x}$

   (b) $f(x) = \dfrac{x^2 - 9}{x^2 - 1}$ (d) $f(x) = \dfrac{2x^3}{x^2 + 1}$

4. Solve the inequalities given below. Write your answers in interval notation. If no solution exists, write "$\varnothing$".

(a) $(x+3)(x-1)<0$ (b) $-3x^2+x\le -4$ (c) $-(x-25)^2>0$

(d) $-x^2+6x-9\le 0$ (e) $(x-4)^2(x+3)<0$ (f) $x^3+10x^2<0$

(g) $x^3\le 9x$ (h) $\dfrac{x-3}{x+1}>0$ (i) $\dfrac{(x+1)^2}{x^2-4}\ge 0$

(j) $\dfrac{x+2}{x-4}\ge 1$ (k) $\dfrac{x+4}{2x+4}\ge 1$

Answers

1a. Degree 2, leading coeff. 1, $x=-1$ mult. 1 and $x=-3$ mult. 1

1b. Degree 2, leading coeff. 4, $x=-5$ mult. 2

1c. Degree 5, leading coeff. 1, $x=2$ mult. 5

1d. Degree 3, leading coeff. 1, $x=2$ mult. 1

1e. Degree 4, leading coeff. 4, $x=-4$ mult. 1 and $x=-3$ mult. 3

1f. Degree 4, leading coeff. -1, $x=0$ mult. 2, $x=5$ mult. 1, $x=-5$ mult. 1

2. (a) I, IV (b) II, IV (c) I, III, (d) I, IV

3a. Domain: $(-\infty,\infty)$. HA: $y=0$. x-int: $(0,0)$. y-int: $(0,0)$

3b. Domain: $x\neq\pm 1$. VA: $x=-1$, $x=1$. HA: $y=1$. x-int: $(-2,0)$, $(2,0)$. y-int: $(0,9)$

3c. Domain: $x\neq -3, x\neq 2$. VA: $x=-3$, $x=2$, HA: $y=1$. x-int: $(-2,0),(3,0)$. y-int: $(0,1)$

3d. Domain: $(-\infty,\infty)$. OA: $y=2x$. x-int: $(0,0)$. y-int: $(0,0)$.

3e. Domain: $(-\infty,0)\cup(0,\infty)$. VA: $x=0$. OA: $y=x+4$. x-int: $(-1,0)$, $(-3,0)$. y-int: None.

4.

(a) $(-3,1)$ (b) $(-\infty,-1]\cup\left[\frac{4}{3},\infty\right)$ (c) $\varnothing$

(d) $(-\infty,\infty)$ (e) $(-\infty,-3)$ (f) $(-\infty,-10)$

(g) $(-\infty,-3]\cup[0,3]$ (h) $(-\infty,-1)\cup(3,\infty)$ (i) $(-\infty,-2)\cup(2,\infty)$

(j) $(4,\infty)$ (k) $[-8,-2)$