

Math 101 Review on Algebraic Expressions

## Learning Goals

- (AL-1) Arithmetic of Real and Complex Numbers:  
I can classify numbers as natural, integer, whole, irrational, rational, and real. I can evaluate and simplify arithmetic expressions without the use of a calculator. I can add, subtract, multiply, rationalize, and divide complex numbers and simplify them into standard form  $a + bi$ .
- (AL-2) Polynomials:  
I can characterize polynomials as monomials, binomials, or trinomials when applicable. I can write polynomials in standard form, identify their degree, and describe the coefficients. I can perform operations on polynomials, including addition, subtraction, multiplication, and (long) division. I can state and apply the formulas for the squares of binomials and the cubes of binomials. I can state and apply the formulas for the difference of two cubes and the sum of two cubes.
- (AL-3) Factoring Polynomials:  
I can factor polynomials using a variety of techniques. I can complete the square of a polynomial expression. I can factor polynomials in the form  $A^3 + B^3$  or  $A^3 - B^3$ .
- (AL-4) Rational Expressions:  
I can perform algebraic operations on rational expressions, including reducing to lowest terms, addition, subtraction, multiplication, and division. I can simplify complex rational expressions.
- (AL-5) Radicals and Exponents:  
I can use the laws of exponents, including those involving roots. I can simplify expressions involving  $n$ th roots. I can simplify expressions involving rational exponents. I can rationalize the denominator or numerator of a given rational expression.

## Review Problems

1. Evaluate each expression and simplify your answer.

$$(a) (7 - 2^2) [7 + (6 + (-3)^2)] \qquad (b) \frac{-7 \cdot 6 - 4 - (-7 - 5)}{-7 \cdot 4 + 7}$$

$$(c) 64 - 6 \cdot 10 + 170 \div (-17) \qquad (d) \left[-\frac{1}{27} - \left(-\frac{5}{4}\right)\right] - \left(\frac{4}{3} - \frac{5}{12}\right)$$

2. Which of the following numbers are integers?  $-75$ ,  $-74.58$ ,  $-8/4$ ,  $-585.22$ .  $11/19$

3. Perform each operation and write as a complex number in standard form  $a + bi$ .

$$(a) \sqrt{-76} \qquad (b) \frac{-4 + \sqrt{-80}}{32} \qquad (c) (-6 - 8i) + (7 + 3i) - 5i$$

$$(d) (2 - 4i)^3 \qquad (e) (7 - 8i)(2 + 5i) \qquad (f) \frac{8 + 5i}{6 - 9i}$$

4. (a) Evaluate the expression  $a^2 - 9a - 6$  when  $a = -3$ .

(b) Evaluate the expression below when  $x = -2$ ,  $y = 3$ , and  $a = -4$ :

$$\frac{\frac{6}{y} - \frac{a}{2}}{\frac{x}{2} + \frac{12}{y}}$$

5. Simplify. Write your answers using only positive exponents and without parentheses.

(a)  $-2(-2w + 2y - 6)$       (b)  $5(y + 5) - 7y$       (c)  $\frac{36uv^4}{45u^3v^3}$

(d)  $(6x^2 - 8x + 5) - (5x^2 + 6x - 4) + (4x^2 + 8x - 3)$       (e)  $\frac{81x^5}{9x^3}$

(f)  $\frac{15b^{-9}a^{-6}}{5a^{-1}b^7}$       (g)  $-5w^3x(2w^2 - 3w^6x^5 + 6x^5)$

(h)  $\frac{m^{-\frac{1}{2}}}{m^{\frac{2}{3}}m^{\frac{3}{2}}}$

6. Find each product and write in standard form.

(a)  $(x - 6)(5x - 2)$       (b)  $(2x + \frac{1}{6})(8x - \frac{1}{3})$       (c)  $(x + 2y)^3$

(d)  $(2x - 4)^3$       (e)  $[(5x - y) + 3z][(5x - y) - 3z]$

7. Simplify each radical expression.

(a)  $\sqrt{180}$       (b)  $\sqrt{12} + 3\sqrt{27} - 13\sqrt{3}$

(c)  $\sqrt{2} \cdot \sqrt{50}$       (d)  $\sqrt{7}(\sqrt{2} + 11)$

(e)  $2\sqrt{3}(\sqrt{15} - \sqrt{7})$       (f)  $\sqrt{\frac{32}{18}}$

8. Simplify each expression. In each problem, assume that each variable represents a positive real number.

(a)  $6x\sqrt{48x} + \sqrt{27x^3}$       (b)  $\sqrt{9w^6}$       (c)  $\sqrt[5]{12v^7}\sqrt[5]{8v^3}$

(d)  $(w^{\frac{7}{10}})^{\frac{4}{5}}$       (e)  $\sqrt{v^7}$       (f)  $\sqrt{45w} - \sqrt{20w}$

(g)  $\sqrt{2z} \cdot \sqrt{11z}$

9. Factor each expression as much as possible.

(a)  $64 - w^2$       (b)  $3x^2 + x - 14$       (c)  $27y^3 + 64$

(d)  $12a - 20a^3$       (e)  $18u^2w^3x^7 + 14u^9w^5$       (f)  $4y^3 + 5y^2 + 24y + 30$

10. Rationalize the denominator and simplify.

(a)  $\frac{9x}{\sqrt{5}}$

(b)  $\frac{\sqrt{2}}{\sqrt{7}}$

(c)  $\frac{3\sqrt{6}-5}{\sqrt{6}+2}$

11. Add or subtract. Factor any polynomial appearing in your answer.

(a)  $\frac{6}{3-7x} - \frac{x}{7x-3}$

(b)  $\frac{5}{7y} + \frac{6}{7y}$

(c)  $\frac{-7}{6v^2x} + \frac{9}{4vx^2}$

(d)  $\frac{4}{x+4} - \frac{5}{3x+12}$

(e)  $\frac{8}{c+5} - \frac{4}{c+5}$

12. Multiply or divide. Factor any polynomial appearing in your answer.

(a)  $\frac{3x-30y}{x+4y} \cdot \frac{x+2y}{x^2-100y^2}$

(b)  $\frac{3x^2-2x-21}{2x^2-9x+7} \cdot \frac{2x-7}{4x-12}$

(c)  $\frac{9x^2-4}{5x^2-3x-14} \div \frac{3x-2}{3x-6}$

(d)  $\frac{\frac{v}{v+3}}{\frac{v+5}{v^2-9}}$

13. Simplify fully. Factor any polynomial appearing in your answer.

(a)  $\frac{-2w^2+32}{w^2-9w+20}$

(b)  $\frac{1-\frac{2}{x+4}}{\frac{4}{x+4}+x}$

(c)  $\frac{\frac{3}{v-1}+3}{\frac{3}{v+1}-3}$

14. Determine the quotient and remainder:

(a)  $3x^3 - x^2 + x - 2$  is divided by  $x + 2$

(b)  $5x^4 - x^2 + x - 2$  is divided by  $x^2 + 2$

15. Determine what number should be added to complete the square of each expression.

(a)  $x^2 - 4x$

(b)  $y^2 + 6y$

(c)  $p^2 + \frac{1}{3}p$

## Answers

1. (a) 66, (b)  $34/21$ , (c)  $-6$ , (d)  $8/27$

2. Integers:  $-75, -8/4$

3. (a)  $2i\sqrt{19}$ , (b)  $-\frac{1}{8} + \frac{\sqrt{5}}{8}i$ , (c)  $1 - 10i$ , (d)  $-88 + 16i$ , (e)  $54 + 19i$ , (f)  $\frac{1}{39} + \frac{34}{39}i$

4. (a) 30, (b)  $4/3$

5. (a)  $4w - 4y + 12$ , (b)  $-2y + 25$ , (c)  $\frac{4v}{5u^2}$ , (d)  $5x^2 - 6x + 6$ , (e)  $9x^2$ , (f)  $\frac{3}{b^{16}a^5}$ , (g)  $-10w^5x + 15w^9x^6 - 30w^3x^6$ , (h)  $\frac{1}{m^{8/3}}$

6. (a)  $5x^2 - 32x + 12$ , (b)  $16x^2 + \frac{2}{3}x - \frac{1}{18}$ , (c)  $x^3 + 6x^2y + 12xy^2 + 8y^3$ , (d)  $8x^3 - 48x^2 + 96x - 64$ , (e)  $25x^2 - 10xy + y^2 - 9z^2$

7. (a)  $6\sqrt{5}$ , (b)  $-2\sqrt{3}$ , (c) 10, (d)  $\sqrt{14} + 11\sqrt{7}$ , (e)  $6\sqrt{5} - 2\sqrt{21}$ , (f)  $4/3$

8. (a)  $27x\sqrt{3x}$ , (b)  $3w^3$ , (c)  $2v^2\sqrt[5]{3}$ , (d)  $w^{14/25}$ , (e)  $v^3\sqrt{v}$ , (f)  $\sqrt{5w}$ , (g)  $z\sqrt{22}$
9. (a)  $(8+w)(8-w)$ , (b)  $(3x+7)(x-2)$ , (c)  $(3y+4)(9y^2-12y+16)$ , (d)  $4a(3-5a^2)$ ,  
(e)  $2u^2w^3(9x^7+7u^7w^2)$ , (f)  $(4y+5)(y^2+6)$
10. (a)  $\frac{9x\sqrt{5}}{5}$ , (b)  $\frac{\sqrt{14}}{7}$ , (c)  $\frac{28-11\sqrt{6}}{2}$
11. (a)  $\frac{-6-x}{7x-3}$ , (b)  $\frac{11}{7y}$ , (c)  $\frac{-14x+27v}{12v^2x^2}$ , (d)  $\frac{7}{3(x+4)}$ , (e)  $\frac{4}{c+5}$
12. (a)  $\frac{3(x+2y)}{(x+4y)(x+10y)}$ , (b)  $\frac{3x+7}{4(x-1)}$ , (c)  $\frac{3(3x+2)}{5x+7}$ , (d)  $\frac{v(v-3)}{v+5}$
13. (a)  $-\frac{2(w+4)}{w-5}$ , (b)  $\frac{1}{x+2}$ , (c)  $\frac{1+v}{1-v}$
14. (a)  $3x^2 - 7x + 15$  remainder:  $-32$ , (b)  $5x^2 - 11$  remainder:  $x + 20$
15. (a)  $+4$ , (b)  $+9$ , (c)  $+\frac{1}{36}$