Arithmetic Combinations of Functions

If f and g are then functions, then:
1) $(f+g)(x) = f(x) + g(x)$

2) $(f-g)(x) = f(x) + g(x)$

3) $(f \bullet g)(x) = f(x) \bullet g(x)$

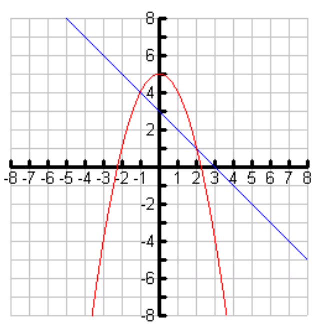
4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

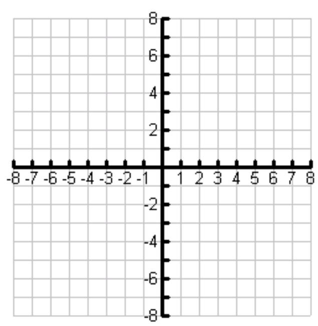
Domain: For $f+g, f-g, f \bullet g$ are those real numbers common to the domains of both f and g or $\{x | x \in \text{domain of } f \text{ and } x \in \text{domain of } g\}$.

Domain for $\frac{f}{g}$ are those real numbers common to the domains of both f and g **and** $g(x) \neq 0$ **or** $\{x | x \in \text{domain of } f \text{ and } x \in \text{domain of } g, g(x) \neq 0\}$.

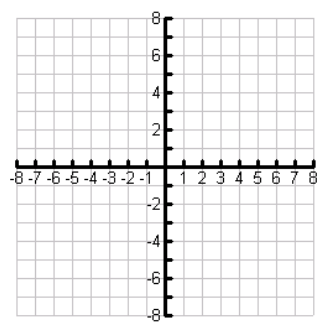
Example: If $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{x}{\sqrt{x+4}}$, find $f+g, F-g, f \bullet g,$ and $\frac{f}{g}$, and give their domains.

Example: The graphs of f and g are given. Sketch the graphs of $f+g$ and $f-g$.





$f+g$



$f-g$

Vertical Compression and Elongation of graphs: $y = c \bullet g(x)$

a) For any constant $c > 0$, graph of $y = c \bullet g(x)$ is the same as $y = g(x)$ with a change in vertical scale.

1) If $c > 1 \rightarrow$ Elongation

2) If $0 < c < 1 \rightarrow$ Vertical Compression

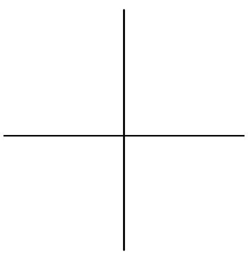
b) The graph of $y = -g(x)$ is a **reflection** about the x-axis of the graph of $y = g(x)$.

1) For $c < -1 \rightarrow$ Elongation and reflection

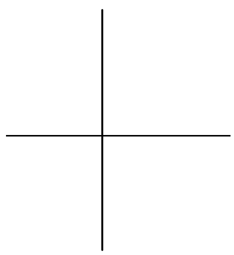
2) For $-1 < c < 0 \rightarrow$ Vertical compression and reflection

Example: Graph the following functions:

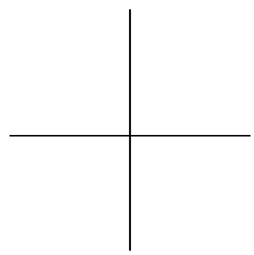
a) $f(x) = 2|x-3|$



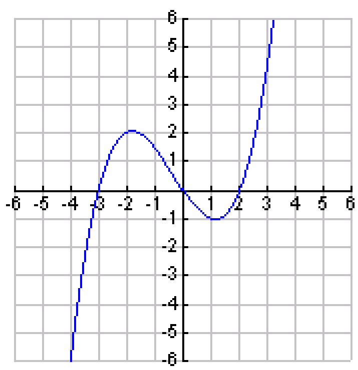
b) $f(x) = \frac{1}{2}|x-3|$



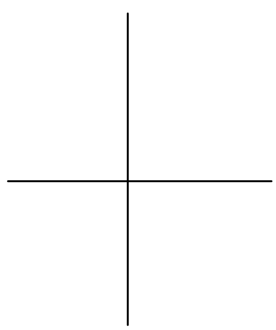
c) $f(x) = -\frac{1}{2}|x-3|$



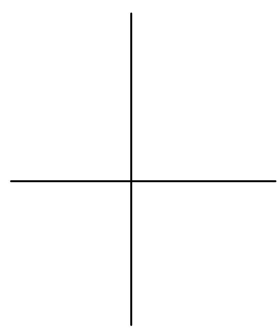
Example: Use the function given below to sketch the graph of the given function.



$y = 2f(x)$



$y = \frac{1}{2}f(x)$



The Graph of the Reciprocal of a function: Let $f(x)=\frac{1}{g(x)}$ be the reciprocal of the function $g(x)$.

1) $f(x)$ is undefined when $g(x)=0$.

2) $f(x)$ and $g(x)$ have the same value when $g(x)=\pm 1$.

3) $f(x)$ and $g(x)$ have the same sign.

4) The magnitude of $f(x)$ is large when the magnitude of $g(x)$ is small.

5) The magnitude of $f(x)$ is small when the magnitude of $g(x)$ is large.

Example: Use the graph of $g(x)=2x+1$ to determine the graph of $f(x)=\frac{1}{g(x)}$.