

# Math 495 Handout: January 10, 2008

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## Course Overview

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- This course fulfills the *Capstone Requirement* for math majors at C of C.

**Question 1:** Why do you think there is a capstone requirement? How should this class be different from other classes you have taken?

- Because of the unusual nature of this class, it will be difficult for me to be very precise about how things will go. I don't know exactly what topics we will cover (good news: you will have some influence on that!) or when they will be covered. Some of the ideas I try will turn out to have been overly ambitious, and other things I expected to be difficult may prove to be quite easy for you. What I can say is that I will aim to ensure that this class involves more independent work, interdisciplinary ideas, and opportunity to practice your communication skills than a typical math course. (See syllabus for more information.)
- The topic I have chosen for the course is one in which I have done most of my research: "Soliton Theory". This is a relatively new area of mathematics, and still an active subject of research. Because of its youth and complexity, it is difficult to give a simple description of the field. I will try to give you an idea now of what it entails and why it is of interest. Then, throughout the course, we will see just a handful of tiny pieces of this large body of work and hopefully learn enough that you can appreciate the "flavor" of this subject.
- In the 19th century, mathematical physics was used to study waves. This allowed, for instance, for the discovery of electro-magnetic waves which we use today for radio broadcasts, cell phone signals, lasers, and microwave ovens. We can categorize waves into two types: linear and nonlinear waves. As it turns out, most of the progress in the 19th century was in work on linear waves. The equations for nonlinear waves, it appeared, were just too difficult to work with.
- Also in the 19th century, other mathematicians worked on "abstract" mathematics like algebras and geometries that may have seemed fanciful at the time.
- In the early 20th century, a revolution in physics demonstrated that some of these supposedly "pure" mathematical results actually had direct application in physics. In particular, quantum mechanics revealed that non-commutative algebras are needed to study particles, and general relativity revealed that the geometry of space is nontrivial and important. QM also demonstrated the importance of waves by showing that all matter is made up of waves. Exactly how and why these waves behave like particles remains somewhat mysterious/controversial today.
- Finally, beginning in the late 1960's, it was discovered that *some* nonlinear wave equations are very special. These "soliton equations" have
  - extra mathematical structure so that they can be solved exactly.
  - wave solutions that behave in a particle-like manner.

By studying these soliton equations, we have not only learned more about waves and particles, but also more about pure math itself. Remarkably, in this interdisciplinary field of research, not only do algebra and geometry get applied to tell us about physics, but the reverse is true as well.

- So, soliton theory combines calculus, algebra and geometry in a single subject. I guess we could say that soliton theory is part of the intersection of these different subjects. For some people, the fact that it is so specialized (involving just a tiny piece of each area) makes it uninteresting. For others, however, the interplay between the different mathematical disciplines and its connection to physics are unbelievably fascinating.

**Question 2:** How do you think *calculus* comes into this?

**Question 3:** What about algebra and geometry?

## Homework

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**Review:** Brush up on any linear algebra and/or differential equations you have taken as we will be using that next time.