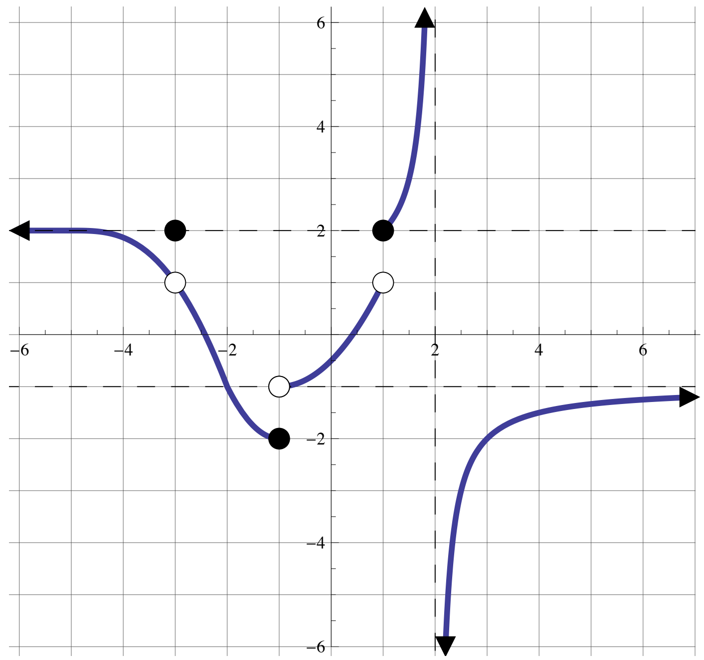
Math 120, Common Final Exam Name:________________________

College of Charleston — Department of Mathematics

Total points: 266, Exam length: 3 hours — April 27, 2011

Instructions: You may *not* use a calculator or any other outside materials.

1. Using the graph of the function f below, find the requested value or limit. (Write "DNE", "$+\infty$" or "$-\infty$" for where appropriate.)



(a) $\lim_{x\to -3} f(x) =$ (b) $f(-3) =$ (c) $\lim_{x\to -2} f(x) =$

(d) $\lim_{x\to -1} f(x) =$ (e) $\lim_{x\to 1^-} f(x) =$ (f) $\lim_{x\to 2^+} f(x) =$

(g) $\lim_{x\to \infty} f(x) =$

2. The point $(1,-1)$ lies on the curve $x^3y^3 + x + y^2 = 1$.
Find the slope of the tangent line to the curve at this point.

3. Compute the value of the following limits using any appropriate methods from this course. If the limit is infinite, write $+\infty$ or $-\infty$ where appropriate.

(a) $\lim_{x\to 4} \frac{\sqrt{2x-4}-2}{x-4}$ (b) $\lim_{x\to 5^+} \frac{2|5-x|}{5-x}$

(c) $\lim_{x\to -\infty} \frac{4x+1}{\sqrt{3x^2+7}}$ (d) $\lim_{x\to \frac{\pi}{2}^+} \tan(x)$

(e) $\lim_{x\to 0} \frac{3x-\sin(x)}{2x}$ (f) $\lim_{x\to 1} \frac{\ln(x^\pi)}{x-1}$

4. Evaluate the following definite integrals. Your answer should be a number written in a simplified form.

(a) $\int_0^9 5\sqrt{x}\,dx$ (b) $\int_{-\pi}^{\pi} \frac{9-\sin(x)}{2}\,dx$ (c) $\int_{49}^{64} \frac{1}{\sqrt{x}(9-\sqrt{x})^3}\,dx$

5. Using only the definition of the derivative and algebraic methods for evaluating limits, find the derivative of the function

$$f(x) = \frac{5x}{3x-2}.$$

6. Given $f(x) = \dfrac{x^2+x-6}{x^2-x-2}$.

   (a) Find $\lim\limits_{x\to 2} f(x)$.

   (b) Does the function have a *vertical asymptote* at $x = 2$? Explain.

7. What are m and b if $y = mx + b$ is the tangent line to the graph $y = x^3 - 5x + 10$ at $x = 2$? (Write your answers as numbers in simplest form.)

   $m =$ __________ $b =$ __________

8. Use properties of integrals to evaluate each of the following given that

$$\int_{-3}^{1} f(x)\,dx = 6, \qquad \int_{1}^{4} f(x)\,dx = -1 \qquad \text{and} \qquad \int_{1}^{4} g(x)\,dx = -4.$$

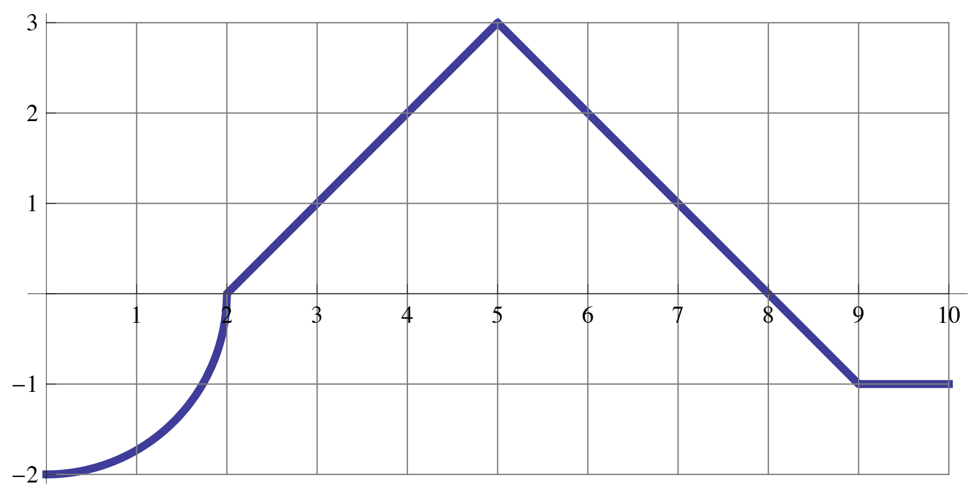
   (a) $\displaystyle\int_{-3}^{4} 4f(x)\,dx$

   (b) $\displaystyle\int_{-3}^{-3} (f(x) - g(x))\,dx$

   (c) $\displaystyle\int_{4}^{1} (3f(x) + g(x))\,dx$

   (d) $\displaystyle\int_{-3}^{1} (3 + f(x))\,dx$

9. Let $P(x) = \displaystyle\int_0^x f(t)\,dt$ where the graph of the function f is shown below. (Note that the graph of f consists of a quarter circle and three straight line segments all of whose endpoints have whole number coordinates.)



Evaluate:

(a) $P(2) =$

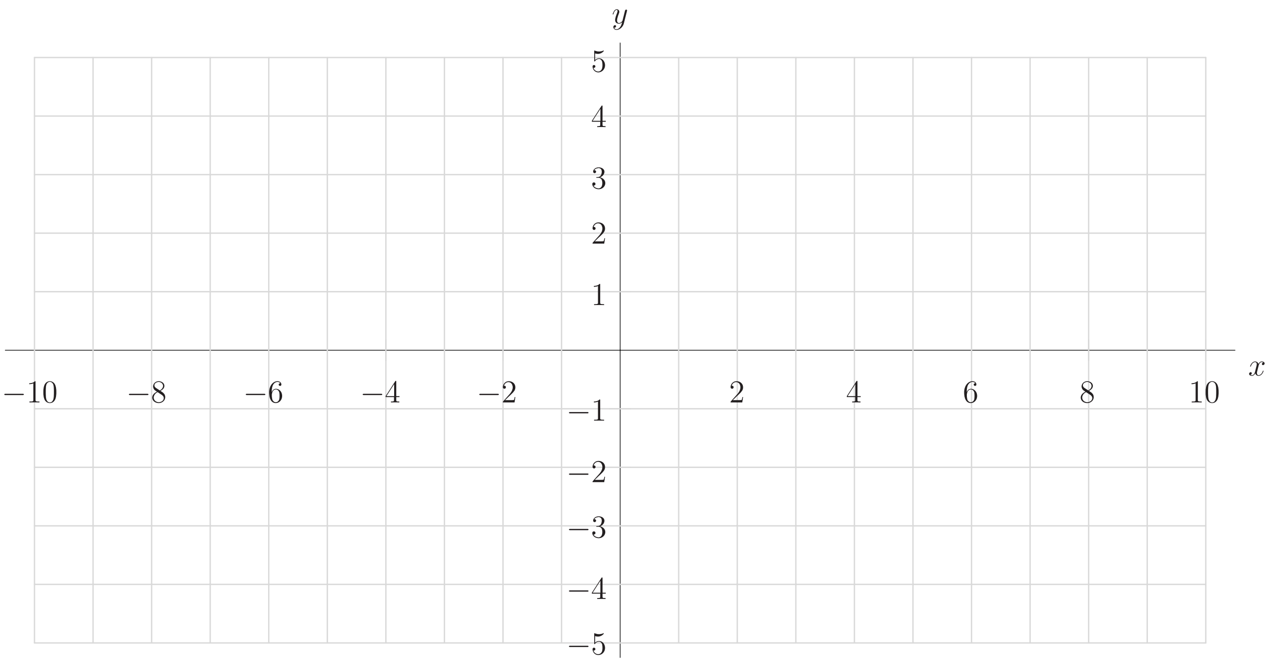
(b) $P(10) =$

(c) $P'(6) =$

10. Find $\dfrac{d}{dx}\left(\displaystyle\int_1^{x^3} 10\sin(t^7)\,dt\right)$. (Simplify your answer.)

11. Find the absolute maximum and minimum values of $f(x) = \dfrac{6x}{9x^2+4}$ on $[0, 2]$.

12. Sketch a possible graph of a *continuous* function f that has all of the following properties:
   (a) $f(-2) = 0$, $f(0) = 2$;
   (b) f' does not exist at $x = 2$;
   (c) $f'(0) = 0$, $f'(x) > 0$ for $x < 0$ and for $2 < x$; $f'(x) < 0$ for $0 < x < 2$;
   (d) $f''(x) > 0$ for $x < -2$; $f''(x) < 0$ for $-2 < x < 2$ and for $2 < x$;
   (e) $\lim\limits_{x\to-\infty} f(x) = -2$, $\lim\limits_{x\to\infty} f(x) = \infty$.



13. Find the requested specific and general antiderivatives.

(a) $\displaystyle\int \left(\frac{1}{x^4} - x^4 - \frac{1}{4}\right) dx$

(b) $\displaystyle\int (x^\pi + \pi^x)\ dx$

(c) $\displaystyle\int \frac{u+1}{\sqrt{u}}\,du$

(d) $\displaystyle\int \frac{x^2}{2x^3+1}\,dx$

(e) $\displaystyle\int 12\sin(x)\cos^5(x)\,dx$

(f) Find an antiderivative, $F(x)$, of $f(x) = -\dfrac{4}{5}\csc^2\left(\dfrac{x}{5}\right)$.

14. (a) If $g(x) = (x^3 - x - 1)e^{x^2}$ then $g'(x) = (2x^4 + x^2 - 2x - 1)e^{x^2}$.
   i. Find $g'(1)$ and $g''(1)$. (Show all work.)
   ii. According to the **Second Derivative Test**, which of these best describes what happens on the graph at $x = 1$?

   I. local minimum
   II. inflection point
   III. local maximum
   IV. cannot tell (test fails)

(b) Answer the following questions about the function $h(x) = \dfrac{3x^4}{4} + x^3 + 1$.

i. On what interval(s) is $h(x)$ *decreasing*?

ii. How many *local extrema* does this function have? Give the exact x and y-coordinates for each local extremum on the graph $y = h(x)$.

15. A particle is moving along a straight line with an acceleration function $a(t) = 2e^t + 3\sin(t)$. Find $s(t)$ given that $v(0) = 3$ and $s(0) = 1$.

16. (a) Find the linear approximation $L(x)$ of $f(x) = \sqrt[3]{1+x}$ at $a = 7$.

(b) Use your result from part (a) to approximate $\sqrt[3]{9}$. Express your answer as a fraction.

17. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs $\$9$ per square meter. Material for the sides costs $\$6$ per square meter. Find the dimensions which minimize the cost of materials for such a container.

18. Find the first derivative of each of the following functions. **Do not simplify.**

(a) $f(x) = \dfrac{2}{x^3} - \dfrac{3}{\sqrt[3]{x^2}} - \ln x$

(b) $f(x) = \dfrac{4x^{1/4}}{1+x^5}$

(c) $f(x) = (\cos(\sin x) + e^x)^9$.

(d) $f(x) = x^3 \sin x$

(e) $f(x) = \ln(\sec x + \tan x)$

(f) $f(x) = \arctan(x^3)$

19. Use a Riemann Sum to approximate

$$\int_1^2 e^{x^2}\, dx$$

using four equal-length subintervals and taking the sample point to be the right-most end-points. (Do not simplify your answer!)

20. Suppose that we know the following information about the functions f and g:

$$f(2) = 4 \quad f'(2) = 3 \quad f(3) = 5 \quad f'(3) = 6 \quad f(4) = 7 \quad f'(4) = 8$$

$$g(2) = 3 \quad g'(2) = 4 \quad g(3) = 9 \quad g'(3) = 10 \quad g(4) = 11 \quad g'(4) = 12.$$

If the functions p, q and h are defined by the formulas

$$p(x) = f(x)g(x) \qquad q(x) = \frac{f(x)}{g(x)} \qquad \text{and} \qquad h(x) = f\left(g(x)\right)$$

evaluate

(a) $p'(2)$

(b) $q'(2)$

(c) $h'(2)$

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1a) 1 1b) 2 1c) −1

1d) DNE 1e) 1 1f) $-\infty$

1g) −1 2) 2

3a) $\frac{1}{2}$ 3b) −2 3c) $-\frac{4}{\sqrt{3}}$

3d) $-\infty$ 3e) 1 3f) π

4a) 90 4b) 9π 4c) $\frac{3}{4}$

5) $-\frac{10}{(3x-2)^2}$

6a) $\frac{5}{3}$ 6b) No, the discontinuity is removable because $\lim_{x\to 2} f(x)$ exists.

7) $m=7$ $b=-6$

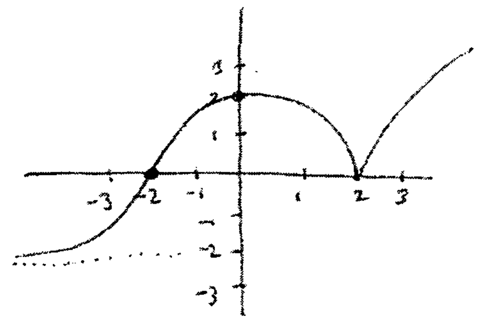
8a) 20 8b) 0 8c) 7 8d) 18

9a) $-\pi$ 9b) $\frac{15}{2}-\pi$ 9c) 2

10) $\overbrace{\left(10\cdot\sin(x^{21})\right)}^{\text{Fund. Thm.}}\cdot\overbrace{(3x^2)}^{\text{Chain}}$
$= 30x^2\sin(x^{21})$

11) Absolute maximum: $f(\frac{2}{3})=\frac{1}{2}$
Absolute minimum: $f(0)=0$.

12)



13a) $-\frac{1}{3x^3}-\frac{1}{5}x^5-\frac{1}{4}x+C$

13b) $\frac{x^{\pi+1}}{\pi+1}+\frac{\pi^x}{\ln\pi}+C$

13c) $\frac{2}{3}u^{3/2}+2u^{1/2}+C$

13d) $\frac{1}{6}\ln|2x^3+1|+C$

13e) $-2\cos^6 x+C$

13f) $4\cot(\frac{x}{5})+C$

14a) i) $g'(1)=0$ $g''(1)=8e$
ii) I local minimum

14b) i) $(-\infty,-1)$
ii) local minimum at $(-1,\frac{3}{4})$

15) $s(t)=2e^t-3\sin t+4t-1$

16) a) $L(x)=2+\frac{1}{12}(x-7)$
b) $\sqrt[3]{9}\approx L(8)=\frac{25}{12}$

17) width $=\sqrt[3]{5}$, length $=2\sqrt[3]{5}$, height $=\sqrt[3]{5}$

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continued.

18) a) $f'(x) = -6x^{-4} + 2x^{-\frac{5}{3}} - \frac{1}{x}$

b) $f'(x) =$

$$\frac{(1+x^5)\cdot(x^{-3/4}) - (4x^{1/4})(5x^4)}{(1+x^5)^2}$$

c) $f'(x) =$

$9(\cos(\sin(x)) + e^x)^8 \cdot (-\sin(\sin(x)) \cdot \cos(x) + e^x)$

d) $f'(x) = (3x^2)\sin(x) + x^3\cos(x)$

e) $f'(x) = \left(\frac{1}{\sec(x) + \tan(x)}\right) \cdot \left(\sec(x)\tan(x) + \sec^2(x)\right)$

f) $f'(x) = \frac{1}{1+x^6} \cdot 3x^2$

19) $\frac{1}{4}\left(e^{25/16} + e^{36/16} + e^{49/16} + e^{4}\right)$

≈ 22.5594

20) a) 25 b) $-7/9$ c) 24