Math 120, Final Exam, Part 1 Name: _____

Total of parts 1&2: 200 pts, 3 hours May 1, 2006

On the first 10 pages of this exam, you may **not** use a calculator or any other outside materials. On the remaining pages, you may use a calculator, but no notes, books, cell phones, etc. You can work on both parts of this exam as long as you want without a calculator, but **you must hand in pages 1-10 of this exam before you get out your calculator**. Exact answers and supporting work are required.

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1. (16 pts) Find the following limits. Write your answer as a number, $+\infty$, $-\infty$, or DNE.

a. $\lim_{x\to\infty}\frac{6x^3+5}{x^3+x}$ b. $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$ c. $\lim_{x\to 0}\frac{\cos(x)-1}{x^2}$ d. $\lim_{x\to 4}\frac{x^2-5x+4}{x-4}$

2. (10 pts) Find the derivative of $f(x)=\frac{3}{x+1}$ using the basic definition of the derivative as a limit of the difference quotient. You can use other methods to check your answer, but to receive credit on this problem, you must compute the derivative using the definition.

3. (6 pts) If $h(x)=f(x)g(x)$ and $F(x)=f(g(x))$,
where $f(2)=3$, $g(2)=5$, $g'(2)=4$, $f'(2)=-2$, and $f'(5)=11$, find
a) $h'(2)$ b) $F'(2)$

4. Find the derivative of each of the following functions. You are not required to simplify your answers.

a. (6 pts) $g(x)=\cos(3x)+\sec(x^2+7)$ b. (6 pts) $y=\left(\ln(3x^2+5)\right)^7$

c. (7 pts) $y=(2x-3)^{4x+1}$

5. (8 pts) Find $\frac{dy}{dx}$ along the curve $y^5+x^2y^3=2+ye^{4x}$.

6. (8 pts) Find the absolute maximum and minimum values of $f(x)=\frac{x}{x^2+4}$ on the interval $[0,4]$.

7. (16 pts) Given the function $f(x)=xe^x$.
a. Does f have any critical numbers? If so, what are the critical numbers of f?
b. Where is the function f increasing and where is it decreasing?
c. Where is the function f concave upward and concave downward?
d. Does f have any inflection points? If so, where are they?

8. (6 pts) On the axes provided, sketch a graph of a function $f(x)$ with all of the following properties:

$f(2)=0$, and $f(0)=3$

$f'(x)>0$ if $x<1$ or $x>1$.

$f''(x)>0$ if $x<1$.

$\lim_{x\to 1^+} f(x)=-\infty$, $\lim_{x\to 1^-} f(x)=+\infty$

$f''(x)<0$ if $x>1$

$\lim_{x\to\pm\infty} f(x)=2$

9. a. (6 pts) Find the linear approximation of $f(x)=\sqrt{x}$ at $a=16$.

b. (2 pts) Use your answer in 9a to approximate $\sqrt{17}$. Give your answer as a rational number in lowest terms.

10. (7 pts) Find all values of c satisfying the conclusion of the Mean Value Theorem for $f(x)=5-\frac{4}{x}$ on the interval [2, 4].

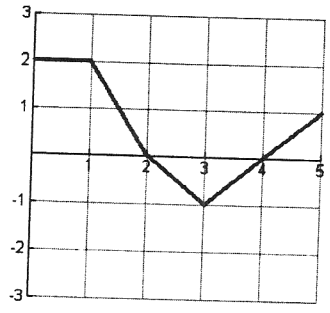
11. Use the properties of integrals to evaluate each of the following given that $\int_{-4}^{3} f(x)dx=-5$, $\int_{3}^{7} f(x)dx=7$, and $\int_{7}^{10} f(x)dx=4$

a. (3 pts) $\int_{7}^{-4} 3f(x)dx$

b. (4 pts) $\int_{3}^{10} (2+f(x))dx$

12. (4 pts) If $G(x)=\int_{1}^{4x}\sqrt{3+2t^2}\,dt$, find $G'(x)$. You needn't simplify your answer.

13. (10 pts) If $g(x)=\int_{0}^{x} f(t)dt$, where $y=f(t)$ is the graph shown



a. Evaluate g(3) = _____ b. Find $g'(3)$ = _____

c. On what interval(s) is $g(x)$ increasing? _____

d. Where does $g(x)$ have a maximum? _____ e. Where does $g(x)$ have a minimum? _____

f. On what interval(s) is $g(x)$ concave down? _____

14. Evaluate the indefinite integrals:

a. (6 pts) $\int(2\cot x\csc x-\frac{4}{x^5}+3\sin x)dx$

b. (5 pts) $\int\frac{x^2}{\sqrt{5+x^3}}dx$

c. (5 pts) $\int\left(3\sec^2(y)\cdot e^{\tan y}\right)dy$

15. (6 pts) If $f'(x)=2x-2\sin x$ and $f(\pi)=0$, find $f(x)$.

16. Evaluate the definite integrals. Simplify your answers.

a. (7 pts) $\int_1^e\left(x^2+\frac{2}{x}\right)dx$

b. (7 pts) $\int_0^{\pi}\frac{\sin x}{(3-\cos x)^2}dx$

c. (7 pts) $\int_0^2 x^2(x^3+1)dx$

End of noncalculator part of exam.

On questions 17 - 21, you may use a calculator, but no notes, books, etc. You can work on both parts of this exam as long as you want without calculator, but you must hand in pages 1-10 of this exam before you get out your calculator. You may not share a calculator with another student.

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17. (10 pts) Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

18. (10 pts) A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of its base is twice the width. Find the dimensions which will minimize the amount of material needed to construct the box.

19. (4 pts) Use a Riemann sum to approximate $\int_0^4 e^x dx$ using two subintervals. Use right endpoints as the sample points. **Sketch your areas on the graph provided. Show your set up and give your result correct to three decimal places.**

20. (4 pts) An object moving along a line has a velocity $v(t)=2t-2$ cm/sec at time t. Its initial position is $s(0)=4\ cm$. Find the **total distance traveled** in the first 4 seconds.

21. (4 pts) The rate at which the world's oil is being consumed is continuously increasing at a rate of $r(t)=32e^{.05t}$, where r is in billions of gallons per year and t is the number of years since 2001 (t = 0 in the year 2001). Find the total amount of oil consumed from 2001 to 2006.

Answers for Math 120 Final Exam May 1, 2006

1. a. 6, b. $\frac{1}{6}$, c. $-\frac{1}{2}$, d. 3

2. $\dfrac{-3}{(x+1)^2}$

3. a. 2, b. 44

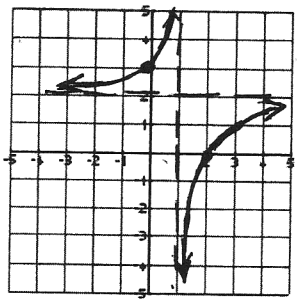
4. a. $-3\sin(3x)+2x\sec(x^2+7)\tan(x^2+7)$, b. $\dfrac{42x(\ln(3x^2+5))^6}{3x^2+5}$

c. $(2x-3)^{4x+1}\left(\dfrac{2(4x+1)}{2x-3}+4\ln(2x-3)\right)$

5. $\dfrac{4e^{4x}y-2xy^3}{5y^4+3x^2y^2-e^{4x}}$

6. abs min is $f(0)=0$, abs max is $f(2)=\frac{1}{4}$

7. a. $x=-1$, b. dec $(-\infty,-1)$, inc $(-1,\infty)$, c. CU $(-2,\infty)$, CD $(-\infty,-2)$, d. $f(-2)=\dfrac{-2}{e^2}$

8. 

9. a. $L(x)=\dfrac{1}{8}x+2$, b. $\dfrac{33}{8}$

10. $2\sqrt{2}$

11. a. -6, b. 25

12. $\sqrt{3+2(4x)^2}\cdot 4$

13. a. 2.5, b. -1, c. (0, 2) (4, 5), d. 2, e. 4, f. (1, 3)

14. a. $-2\csc x+x^{-4}-3\cos x+C$, b. $\dfrac{2}{3}(5+x^3)^{\frac{1}{2}}+C$, c. $3e^{\tan y}+C$

15. $x^2+2\cos x+2-\pi^2$

16. a. $\dfrac{e^3+5}{3}$, b. $\dfrac{1}{4}$, c. $\dfrac{40}{3}$

17. 65 mph

18. $\sqrt[3]{\dfrac{15}{2}}\approx 1.957$ m, 1.305 m

19. 123.974

20. 10 cm

21. 181.776 billion gallons