1. For each of the following problems, find the indicated derivative.

a. \((4 \text{ pts}) f(x) = 12x^{\frac{3}{2}} - 30x^{\frac{1}{2}} + \frac{1}{x^5}\), find \(f'(x)\).
\[
f'(x) = \frac{8}{\sqrt{x}} - \frac{15}{\sqrt{x}} - \frac{5}{x^6}
\]

b. \((5 \text{ pts}) g(x) = \frac{6\sqrt{x}}{3x-4}\), find \(g'(x)\).
\[
g'(x) = \frac{(3x-4)(3x^{-\frac{1}{2}}) - 6\sqrt{x}}{(3x-4)^2}
\]
\[
= \frac{9\sqrt{x} - 12x^{-\frac{1}{2}} - 6\sqrt{x}}{(3x-4)^2}
\]
\[
= \frac{-9x - 12}{\sqrt{x}(3x-4)^2}
\]

c. \((5 \text{ pts}) h(x) = -7x(x^4 - 2)^3\), find \(\frac{dh}{dx}\).
\[
h'(x) = -7x\left[3(x^4 - 2)^2(4x^3)\right]
\]
\[
\cdot -7(x^4 - 2)^2
\]
\[
= -84x^4(x^4 - 2)^2 - 7(x^4 - 2)^3
\]
\[
= (x^4 - 2)^2 \left[ -84x^4 - 7x^4 + 14 \right]
\]
\[
= (x^4 - 2)^2 \left[ -91x^4 + 14 \right]
\]

d. \((4 \text{ pts}) f(x) = e^{1-x^2}\), find \(f''(x)\).
\[
f'(x) = e^{1-x^2}(-2x)
\]
\[
f''(x) = -2e^{1-x^2} - 2xe^{1-x^2}(-2x)
\]
\[
= -2e^{1-x^2} + 4xe^{1-x^2}
\]
\[
= e^{1-x^2}(4x^2 - 2)
\]

e. \((4 \text{ pts}) f(x, y) = \ln(x^2 + 3xy)\), find \(\frac{\partial f}{\partial x}\).
\[
\text{NOT ON EXAM}
\]

f. \((4 \text{ pts}) f(x, y) = 5x^3 y - 3x^3 y^2 - 13\), find \(f_{yy}\).
\[
\text{NOT ON EXAM}
\]
2. For each of the following problems, find the indicated integral.
   a. \((5 \text{ pts}) \int (y^3 + 7y^2 - 2y) \, dy = \frac{y^4}{4} + \frac{7y^3}{3} - y^2 + C\)

   b. \((4 \text{ pts}) \int (e^x - 2\sqrt{x}) \, dx = e^x - \frac{2x^{3/2}}{3/2} + C = e^x - \frac{4x^{3/2}}{3} + C\)

   c. \((4 \text{ pts}) \int_{1}^{3} \frac{5}{x} \, dx = 5 \ln x \bigg|_{1}^{3} = 5 \ln 3 - 5 \ln 1 = 5 \ln 3\)

3. \(6 \text{ pts}) \) Find the equation of the tangent line for the function \(f(x) = (x^5 - 5)(x^3 - x - 1)\) at \(x = 0\).
   \(f(0) = (-5)(-1) = 5\)
   \(y - 5 = 5(x - 0)\)
   \(y = 5x + 5\)

4. \(7 \text{ pts}) \) One thousand dollars is deposited in a savings account where the interest is compounded continuously. After 4 years, the balance will be $1366.15. When will the balance be $1870.00?
   \[A = Pe^{rt}\]
   \[1366.15 = 1000e^{4r}\]
   \[\ln \frac{1366.15}{1000} = 4r\]
   \[0.31997 \approx 4r\]
   \[0.078 \approx r\]
   \[A = 1000e^{0.078t}\]
   \[1870 = 1000e^{0.078t}\]
   \[\ln \frac{1870}{1000} = 0.078t\]
   \[0.31997 \approx 0.078t\]
   \[t \approx 8 \text{ years}\]
5. (9 pts) Managers at an electronics manufacturer estimate that to sell $x$ of a particular computer each day, the price of a computer should be

$$D(x) = 2000 - 12x \text{ dollars}$$
$$R(x) = 2000x - 12x^2$$

The total cost to produce all $x$ computers to sell in a day is

$$C(x) = 1275 + 800x \text{ dollars}$$

Express the profit as a function of $x$ and use methods of calculus to determine the manufacturer's maximum daily profit to the nearest dollar. Remember to justify your conclusions and show all your work.

$$P(x) = R(x) - C(x) = 2000x - 12x^2 - (1275 + 800x)$$
$$= -12x^2 + 1200x - 1275$$
$$P'(x) = -24x + 1200 = 0$$
$$x = 50$$
$$P''(x) = -24 < 0 \Rightarrow x = 50 \text{ is a maximum}$$

$$P(50) = -12(50)^2 + 1200(50) - 1275 = 28,725$$

The maximum daily profit is $28,725.

6. (9 pts) Find all critical points for the function

$$f(x) = 2x^3 - 9x^2 - 60x + 7$$

Give an ordered pair of coordinates $(x, y)$ for each critical point and classify each critical point as a local maximum, a local minimum, or neither. Use the first derivative test or second derivative test. Remember to justify your conclusions and show all your work.

$$f'(x) = 6x^2 - 18x - 60 = 0$$
$$6(x^2 - 3x - 10) = 0$$
$$6(x - 5)(x + 2) = 0$$
$$x = 5, -2$$

$$f'(-2) = 48 \quad f'(-3) = -2$$
$$f'(5) = -60 \quad f'(6) = 48$$

$$f(-2) = 75$$
$$f(5) = -268$$

Maximum at $(-2, 75)$.
Minimum at $(5, -268)$. 
7. (8 pts) The cost of producing $x$ compact refrigerators is given by $C(x) = 2880 + 35x + 0.2x^2$ dollars. Find the value of $x$ that minimizes the average cost function if between 1 and 150 refrigerators are produced.

$$
\bar{C}(x) = \frac{2880}{x} + 35 + 0.2x
$$

$$
\bar{C}'(x) = -\frac{2880}{x^2} + 0.2 = 0
$$

$$
-\frac{2880}{x^2} = -0.2
$$

$$
2880 = 0.2x^2
$$

$$
14400 = x^2
$$

$$
x = 120
$$

The average cost function is minimized at 120 refrigerators.

8. (7 pts) A sporting goods store sells 200 baseball gloves per month at $36 each. The owner estimates that for each $2 increase in price, he will sell 5 fewer gloves. Find the price that will maximize revenue.

$$
R(x) = (200 - 5x)(36 + 2x)
$$

$$
R'(x) = (200 - 5x)(2) + (36 + 2x)(-5)
$$

$$
= 400 - 10x - 180 - 10x
$$

$$
= 220 - 20x = 0
$$

$$
x = 11
$$

$$
R''(x) = -20 < 0 \Rightarrow \text{Maximum}
$$

$$
p = 36 + 2(11) = 36 + 22 = 58
$$

The revenue will be maximized at $58.
9. (9 pts) An automobile agency sells two car models of a car. The manager has determined that the annual profit is estimated by $P(x, y) = -0.1x^2 - 0.2y^2 + 6x + 10y - 160$ in thousands of dollars. Find the number of each model that should be sold to maximize profit and find the maximum profit.

10. (6 pts) Find the area of the region bounded by the graphs of the following equations:

$$y = x + 1$$
$$y = x^2 + x$$

$$A = \int_{-1}^{1} [(x+1) - (x^2 + x)] \, dx$$

$$A = \int_{-1}^{1} (-x^2 + 1) \, dx$$

$$\frac{-x^3}{3} + x \bigg|_{-1}^{1} = \left(\frac{-1}{3} + 1\right) - \left(\frac{1}{3} - 1\right)$$

$$= \frac{2}{3} - \left(\frac{-2}{3}\right) = \frac{4}{3}$$