College of Charleston
Department of Mathematics

Name:

Examination in: Discrete Mathematics

Math Course Number	Math 207
Examination Date	3-23-2006
Examination Time	4:00-5:15

Total number of problems: 7

Professor: Ben Cox

Proctor: Ben Cox **Phone number:** 953-5715

Results available by: March 28 **in:** Maybank 219

Permitted aids: Proctor. No calculators are to be used on this exam.

Show all work to receive full credit.

	Score
1	
2	
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Total	

1. a) Using the Euclidean Algorithm find the greatest common divisor of 462 and 1111.

Sln: $1111 = 2 \cdot 462 + 187$, $462 = 2 \cdot 187 + 88$, $187 = 2 \cdot 88 + 11$, $88 = 8 \cdot 11$. Thus $\gcd(462, 1111) = 11$.

b) Find the prime factorization of the the following numbers: 462 and 1111.

$$1111 = 11 \cdot 101, \quad 462 = 11 \cdot 42 = 2 \cdot 3 \cdot 7 \cdot 11.$$

c) Write the greatest common divisor of 462 and 1111 as a linear combination of 462 and 1111.

Using the first part of this problem we get $11 = 187 - 2 \cdot 88 = 187 - 2(462 - 2 \cdot 187) = 5 \cdot 187 - 2 \cdot 462 = 5(1111 - 2 \cdot 462) - 2 \cdot 462 = 5 \cdot 1111 - 12 \cdot 462$

2. Let m be a positive integer, and let a, b, and c be integers. Show that if $a \equiv b \mod m$, then $ac \equiv bc \mod m$.

Proof:

$$\begin{aligned} a \equiv b \mod m &\leftrightarrow m | a - b \\ &\rightarrow \exists k \in \mathbb{Z}\,(mk = a - b) \\ &\rightarrow \exists k \in \mathbb{Z}\,(mk = a - c - b + c = a - c - (b - c)) \\ &\rightarrow \exists k \in \mathbb{Z}\,(m | a - c - (b - c)) \\ &\rightarrow a - c \equiv b - c \mod m. \end{aligned}$$

3. Find an integer k such that $0 \le k < 13$ such that $3^{18} \equiv k \mod 13$.

Sln:

$$\begin{aligned} 3^3 &\equiv 27 \equiv 1 \mod 13 \\ 3^{18} &= (3^3)^6 \equiv 1^6 \equiv 1 \mod 13. \end{aligned}$$

4. Let $A = \begin{pmatrix} 1 & -1 & -1 \\ 2 & 0 & -3 \\ 2 & 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & 5 \\ 2 & 1 & 7 \\ 3 & 2 & -2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & -3 & -1 \end{pmatrix}$.

a) Find $A + B$.

Sln:

$$A + B = \begin{pmatrix} 2 & 1 & 5 \\ 2 & 1 & 7 \\ 3 & 2 & -2 \end{pmatrix}$$

b) Find CA.

Sln:

$$CA = \begin{pmatrix} -7 & -4 & 7 \end{pmatrix}.$$

c) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$, find $C \bigodot A$.

$$A \odot C = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 0) \\ (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) \\ (1 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 1) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}.$$

5. a) What is the binary expansion of $(1010101)_2 + (1010111)_2$?

b) What is the binary expansion of $(10001)_2 \cdot (10111)_2$?

Sln: $(1010101)_2 + (1010111)_2 = (10101100)_2$, $(10001)_2 \cdot (10111)_2 = (1100001111)_2$

6. a) Find the multiplicative inverse of 3 modulo 7.

b) Solve $3x \equiv 2 \mod 7$.

Sln: First we find the multiplicative inverse of 3 modulo 7. We can do this in at least two ways. One way is to plug into $3x \equiv 1 \mod 7$ the numbers $x = 1, 2, \ldots 6$ into this equation and then find that $3 \cdot 5 \equiv 1 \mod 7$. The other is to use Bezout's lemma to find $7 - 3 \cdot 2 = 1$ so that $-2 \cdot 3 \equiv 1 \mod 7$. Now $-2 \equiv 5 \mod 7$ so that

$$3x \equiv 2 \mod 7 \qquad x = 5 \cdot 3x \equiv 5 \cdot 2 = 10 \equiv 3 \mod 7.$$

7. Give as good a big-O estimate as possible for each of the functions

a) $2n^3 + 3n^2 \log n$.

Sln: $O(n^3)$.

b) $(2n^3 + 3n^2 \log n)(5 + \log n) + (\pi \log n + 2)(n^5 + 3)$

Sln: $O(n^5 \log n)$.

c) $(2n^3 + 2^n)(3^n + n^3)$

Sln: $O(6^n)$ as 2^n and 3^n grow faster than n^k for any fixed k.

d) $(2n^3 + 2^n)(n! + 7^n)$

Sln: $O(2^n n!)$ or $O((n!)^2)$ are both o.k.