College of Charleston
Department of Mathematics

Name:

Examination in: Discrete Mathematics

Math Course Number	Math 207
Examination Date	3-22-2004
Examination Time	4:00-5:15

Total number of problems: 8

Professor: Ben Cox

Proctor: Ben Cox **Phone number:** 953-5715

Results available by: March 25 **in:** Maybank 219

Permitted aids: Proctor

Show all work to receive full credit.

	Score
1	
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Total	

1. Let $f(n) = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3$ for $n \in \mathbb{Z}^+$.

a) Is f a one-to-one function from the set of positive integers to the set of positive integers? Is f an onto function from the set of positive integers to the set of positive integers? Explain the reason behind your answers.

b) Show that $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3$ is $O(n^4)$. Be sure to specify the values of the witnesses C and k.

2. Describe an algorithm for finding the second largest integer in a finite sequence of distinct integers.

3. Let m be a positive integer, and let a, b, and c be integers. Show that if $a \equiv b \mod m$, then $a - c \equiv b - c \mod m$.

4. Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 0 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & -3 \\ 2 & -1 \end{pmatrix}$.

a) Find $A + B$.

b) Find AC.

c) If $A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, find $A \odot C$.

5. Find the prime factorization of 111111.

6. Find an integer k such that $0 \leq k < 17$ such that $2^{18} \equiv k \mod 17$.

7. Use the Euclidean algorithm to find

a) $\gcd(201, 302)$,

b) $\gcd(144, 233)$.

8. a) What is the binary expansion of $(1010101)_2 + (1111111)_2$?

b) What is the binary expansion of $(10101)_2 \cdot (11111)_2$?