College of Charleston
Department of Mathematics

Name:

Examination in: Discrete Mathematics

Math Course Number	Math 207
Examination Date	4-30-2005
Examination Time	4:00-7:00

Total number of problems: 17

Professor: Ben Cox

Proctor: Ben Cox

Phone number: 953-5715

Results available by: May 3, 2005

in: Maybank 219

Permitted aids: Proctor

Show all work to receive full credit.

	Score
1	
2	
3	
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7	
Total	

1. Determine whether $(p \to q) \wedge (\neg p \to q) \equiv q$.

2. Write the contrapositive, converse, and inverse of the following: If you try hard, then you will win.

3. Suppose $P(x, y)$ is the statement "x and y are real numbers such that $x+2y = 5$". Determine whether the following propositions are true.

a) $\forall x \exists y \, P(x, y)$

b) $\exists x \forall y \, P(x, y)$

4. Prove or disprove that if A and B are sets then $A \cap (A \cup B) = A$.

5. Use the definition of big-oh to show that $f(n) = 3n^2 + 8n + 7$ is $O(n^2)$. Find the witnesses C and k.

6. Describe an algorithm that takes a list of n integers $a_1, \ldots, a_n$ $(n \geq 2)$ and finds the second-greatest integer in the list.

7. Find the prime factorization of 16575.

8. a) Prove or disprove: If $a \equiv b \mod 5$, where a and b are integers, then $a^2 \equiv b^2 \mod 5$.

b) Prove or disprove: If $a^2 \equiv b^2 \mod 5$, where a and b are integers, then $a \equiv b \mod 5$.

9. Use mathematical induction to prove that $n! \geq 2^{n-1}$ for any positive integer $n \geq 1$.

10. Give a recursive definition with initial condition(s) for the set of numbers a_n in $\{3, 7, 11, 15, 19, 23, \ldots\}$.

11. Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$. Find $\mathbf{AB}$ and $\mathbf{BA}$. Are they equal?

12. Find each of the following quantities:

a) $C(5, 4)$ which is the same as $\binom{5}{4}$.

b) $C(5, 0)$ which is the same as $\binom{5}{0}$.

c) $P(5, 1)$.

d) $P(5, 5)$.

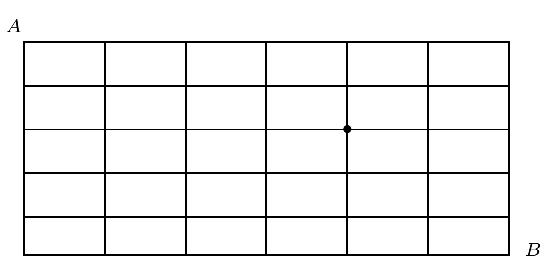
13. a) Show that the relation $R = \{(x, y) \mid x - y \text{ is an even integer}\}$ is an equivalence relation on the set of real numbers.

b) What are the equivalence classes of 1 and 1/2 with respect to R

14. a) Does the collection of sets $\{1, 2, 5\}$, $\{2, 4, 7\}$ and $\{3, 5, 6\}$ form a partition of $\{1, 2, 3, 4, 5, 6, 7\}$?

b) Does the collection of sets $\{1, 4, 7, 8\}$, $\{2, 3\}$ and $\{5, 6\}$ form a partition of $\{1, 2, 3, 4, 5, 6, 7, 8\}$?

15. Consider the grid below.

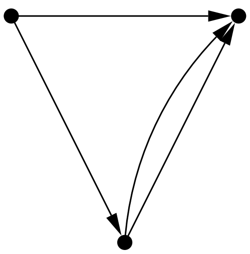


Suppose that starting at point labeled A an ant can go one step down or one step to the right at each move. This is continued until the point labeled B is reached.

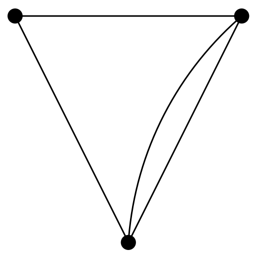
a) How many different paths from A to B are possible?

b) How many different paths are there from A to B that go through the dot indicated above?

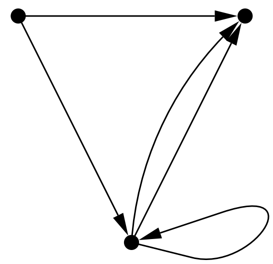
16. Consider the graphs below:



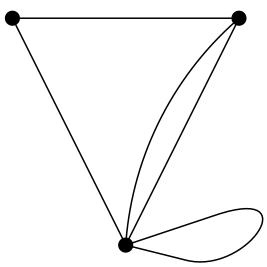
a)



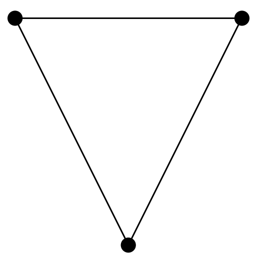
b)



c)



d)



e)

Determine which is a simple graph, a multigraph (and not a simple graph), a pseudo-graph (and not a simple graph), a directed graph, or a directed multigraph (and not a directed graph).

17. a) Use a table to express the values of the Boolean function $f(x, y, z) = x\bar{y}\bar{z} + y\bar{z}\bar{x} + z\bar{x}\bar{y}$.

b) Determine whether $f(x, y, z) = f(x, z, y)$.