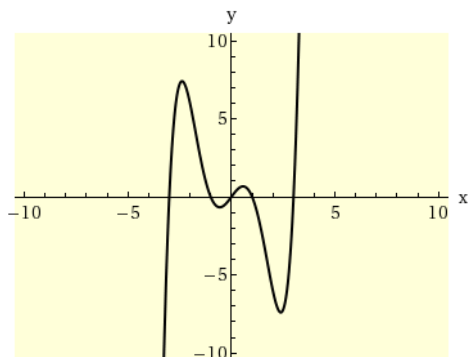


MATH 111 CHAPTER 3 Review (sections 3.1-3.5)

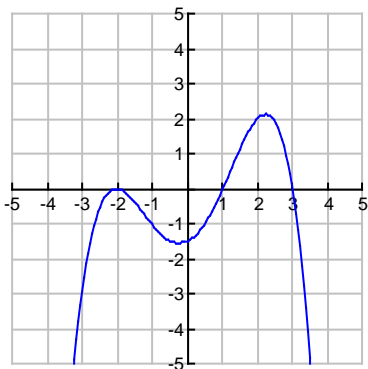
1. Terms to know: polynomial function, degree, leading coefficient, zeros, roots, factors, end behavior of polynomial functions, turning point, repeated zero of multiplicity m , dividend, divisor, quotient, remainder, complete factorization, long division, synthetic division, complex number, real and imaginary parts, complex conjugate, rational zero, complex zero, irrational zero, rational functions, domain and range of the rational functions, vertical, horizontal, slant asymptotes, hole in the graph of rational function, point of crossing an asymptote.
2. Be able to find the equation of the polynomial function from the given graph and from the given conditions (zeros, intercepts, points, end behavior). (sec. 3.1)
3. Be able to sketch the graph of the polynomial function based on the given conditions (zeros, multiplicity, end behavior, etc). (sec. 3.1)
4. Know and be able to use long division, synthetic division, and the Division Algorithm. (sec. 3.2)
5. Be able to divide a polynomial by another polynomial using the long division and synthetic division. (sec. 3.2)
6. Theorem to know: Intermediate Value Theorem, Factor Theorem, Remainder Theorem, Fundamental Theorem of Algebra, Complete Factorization Theorem, Conjugate Zeros Theorem, Rational Zeros Theorem. (sec. 3.1, 3.2, 3.3, 3.4)
7. Be able to find all exact zeros (rational, irrational and complex) of a function both algebraically and graphically. (sec.3.3, 3.4)
8. Be able to recognize when a function has complex roots both algebraically and graphically, and find all exact complex roots of the polynomial. (sec. 3.3)
9. Be able to give the complete factorization of a relatively simple polynomial function of degree 3, 4 and 5. (sec. 3.3, 3.4)
10. Be able to verify if the indicated complex number is a zero of the polynomial function and use Conjugate Zeros Theorem to find ALL other zeros of the polynomial function. (sec.3.3)
11. Be able to simplify and write a complex number in the standard form $a + bi$. (sec. 3.3)
12. Be able to find the domain, range, x- and y-intercepts, all asymptotes and sketch the graph of a rational function. (sec. 3.5)
13. Know applications for polynomial and rational functions. (sec. 3.1, 3.5)
14. Be able to recognize when the rational function has a hole and find the x- and y- coordinates of the hole. (sec. 3.5)
15. Be able to check if the rational function crosses a horizontal and/or slant asymptote and be able to find a point of crossing. (sec. 3.5)
16. Any handouts given in class, any class discussions. (sec. 3.5)

Partial Review Exercises

1. Find an equation of the polynomial function below that passes through (2,-6). SLO 2



2. Determine the polynomial of degree 4 whose graph is shown in the figure. **SLO 2**



3. **SLO 7, 8, 10** a). If $x = 2$ and $x = -2$ are zeros of $f(x) = x^4 + 2x^3 - 2x^2 - 8x - 8$, find all zeros of the polynomial.

b). If $x = 2i$ is a zero of $f(x) = x^4 + 2x^2 - 8$, find all zeros of the polynomial.

c). If $x = 1$ is a zero of $f(x) = x^3 + x^2 - 3x + 1$, find all zeros of the polynomial.

d) If $x = -\frac{1}{2}$ is a zero of $f(x) = 2x^3 - x^2 + x + 1$, find all zeros of the polynomial.

4. Use the Remainder Theorem to find the remainder r when $f(x) = 4x^3 - x^2 + 4$ is divided by $x - 2$. **SLO 4, 6**.

5. If $x = 2/3$ is a zero of the polynomial $f(x) = 3x^3 - 8x^2 - 8x + 8$, find the complete factorization of $f(x)$. **SLO 9**

6. Write each expression in standard complex form: $a + bi$ a) $\frac{8-i}{2+i}$; b) $\sqrt{-8} + \sqrt{-18}$; c) $i(9+12i) - i(1-8i)$.

SLO 11

7. Find a polynomial of lowest degree having only real coefficients and zeros at 3 and $2 + i$. Write your answer in expanded form. **SLO 2**

8. Find a polynomial of lowest degree having only real coefficients and zeros at $x = 1+2i$ and $x = 2$ of multiplicity 2. Write your answer in expanded form. **SLO 2**

9. Determine the end behavior of $P(x) = -x^4 - 5x^2 + 8x + 8$; $y \rightarrow a$ as $x \rightarrow \infty$ and $y \rightarrow b$ as $x \rightarrow -\infty$. **SLO 3**

10. If $x = \frac{1}{2}i$ is a zero of the polynomial $f(x) = 12x^3 + 8x^2 + 3x + 2$, find all zeros. **SLO 8, 10**

11. Sketch the graph of polynomial $P(x)$ that has zeros of multiplicity one at $x = 0$ and $x = 1$, has a zero of multiplicity three at $x = -3$, and satisfies $P(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ and $P(x) \rightarrow \infty$ as $x \rightarrow \infty$. **SLO 3**

12. Graph a) $f(x) = 2x^4 - 8x^3$; b) $f(x) = -x^3 + x^2 + 2x$; c) $p(x) = x^4 - 16x^2$. **SLO 3**

13. Use synthetic division to find the quotient $q(x)$ and remainder $r(x)$ when $f(x) = x^4 + 15$ is divided by the linear polynomial $p = x + 2$. **SLO 5**

14. Use synthetic division and the Remainder Theorem to find $f(c)$ for

$f(x) = 13x^4 - 20x^3 + 42x^2 - 22x + 13$ and $c = 1$. **SLO 4, 5, 6**

15. Use long division to find the value of k such that $f(x) = x^4 + x^3 + 4x^2 + kx - 5$ is divisible by $d(x) = x^2 - 1$. **SLO 4, 5**

16. Solve for x : a) $7x^2 - 2x + 2 = 0$; b) $x^2 + 9 = 0$. SLO 8

17. Use synthetic division to find a value of k such that $f(x) = kx^4 + 5x^2 + 3k$ is divisible by $d(x) = x - 1$. SLO 4, 5

18. Sketch the graph of

a) $g(x) = \frac{x+1}{x-2}$; b) $g(x) = \frac{3x^2+7x-6}{x^2-x-6}$; c) $f(x) = \frac{x^2-2x-3}{x-1}$. SLO 12

19. Graph a) $y = \frac{x^2}{42x - x^2 - x^3}$; b) $y = \frac{x^2 - 4}{x^2 - 2x}$. SLO 12, 14

20. Find the point where the graph of $f(x) = \frac{5x^2}{x^2 + x + 1}$ crosses its horizontal asymptote. SLO 15

21. Find all asymptotes of a) $y = \frac{x^2 - 8x - 9}{x - 1}$, b) $y = \frac{2x^2 + x - 1}{x^2 - x - 2}$. SLO 12

22. An open box is to be made from a square piece of material 32 centimeters on a side by cutting equal squares (x centimeters) from the corners and turning up the sides. Determine the volume of the box as the function of x . SLO 13

23. Find all exact zeros of the following SLO 7

a). $f(x) = x^4 + 2x^3 - 2x^2 - 8x - 8$

b). $f(x) = x^4 + 2x^2 - 8$

c). $f(x) = x^3 + x^2 - 3x + 1$

d) $f(x) = 2x^3 - x^2 + x + 1$

e) $f(x) = x^3 + 5x^2 - 4x - 2$

24. Find a polynomial of degree 2 with real coefficients and zero $x = 1 + i$ that passes through $(1, 5)$. SLO 9

25. Give complete factorization for the given polynomial: SLO 9

a) $p(x) = x^2 - 3x - 6$

b) $p(x) = x^5 + 4x^4 - 6x^3 - 24x^2 + 5x + 20$

c) $p(x) = 8x^3 - x^2 + x + 7$.

Answers:

1. $y = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}$.

2.) $P(x) = -\frac{1}{8}(x^4 - 9x^2 - 4x + 12)$.

3. a). $x = -2, x = 2, x = -1 \pm i$; b). $x = \pm\sqrt{2}$; $x = \pm 2i$; c). $x = 1; x = -1 \pm \sqrt{2}$,

d) $x = -\frac{1}{2}, x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$; e) $x = 1, x = -3 \pm \sqrt{7}$.

4. 32.

5. $3(x - \frac{2}{3})(x - 1 - \sqrt{5})(x - 1 + \sqrt{5})$

6.a) $3-2i$; b) $5\sqrt{2}i$; c) $-20+8i$.

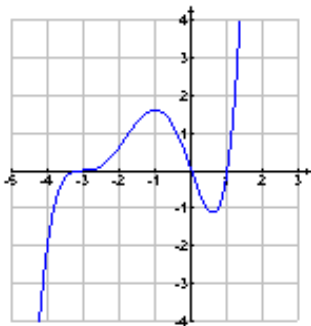
7. $x^3 - 7x^2 + 17x - 15$

8. $x^4 - 6x^3 + 17x^2 - 28x + 20$

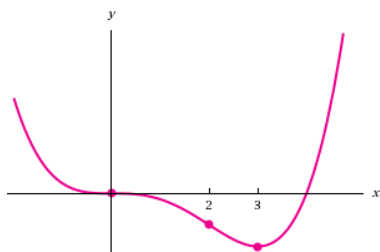
9. $a = -\infty, b = -\infty$.

10. $x = -\frac{2}{3}; x = \pm \frac{1}{2}i$.

11.



12. a)



13.

$q(x) = x^3 - 2x^2 + 4x - 8; r(x) = 31$.

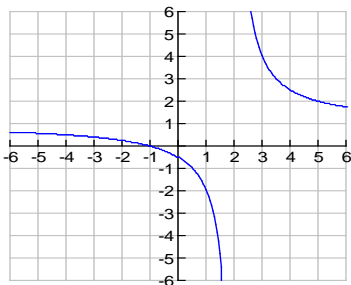
14. $f(1) = 26$.

15. $k = -1$.

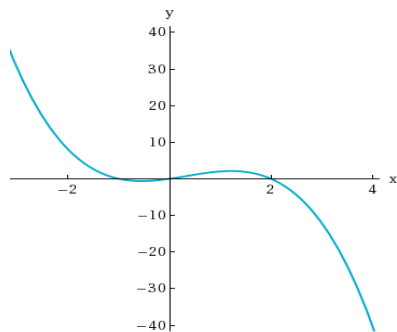
16. a) $x = \frac{1}{7} \pm \frac{i\sqrt{13}}{7}$; b) $x = \pm 3i$.

17. $k = -\frac{5}{4}$.

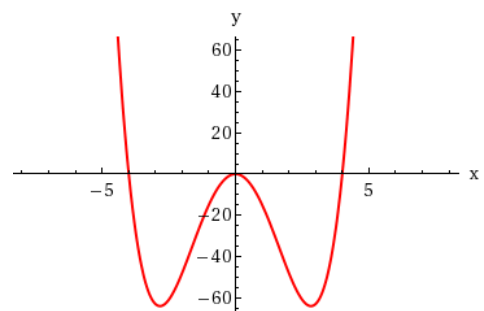
18. a)



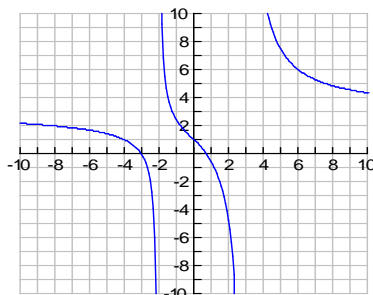
b)



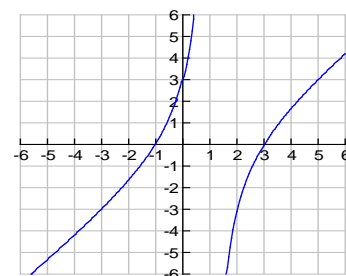
c)



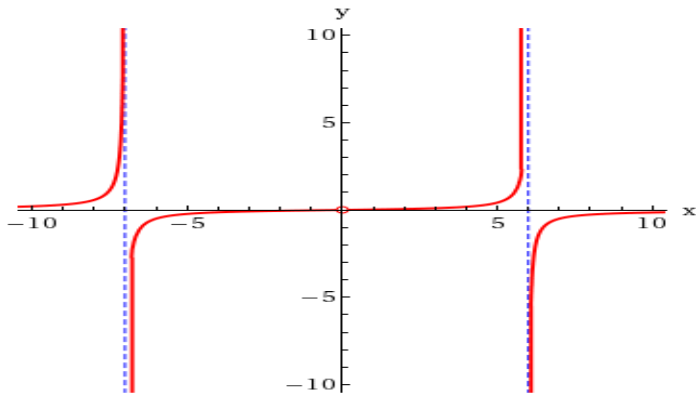
b)



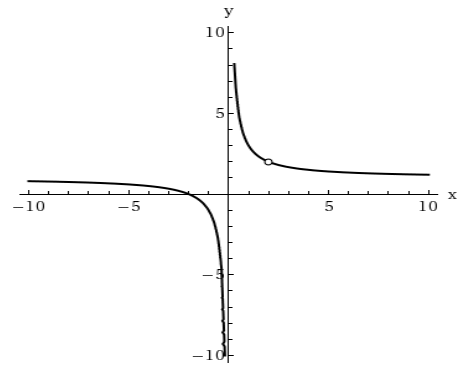
c)



19. a) hole (0,0).



b) hole (2,2)



20. (-1,5).

21. a) VA: $X = 1$; SA: $y = x - 7$; b) VA $x = 2$, HA $y = 2$.

22. $V(x) = x(32 - 2x)^2$.

23. . a). $x = -2, x = 2, x = -1 \pm i$; b). $x = \pm\sqrt{2}; x = \pm 2i$; c). $x = 1; x = -1 \pm \sqrt{2}$,

d) $x = -\frac{1}{2}, x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$; e) $x = 1, x = -3 \pm \sqrt{7}$.

24. $f(x) = 5x^2 - 10x + 10$.

25. a) $p = (x - \frac{3}{2} - \frac{\sqrt{33}}{2})(x - \frac{3}{2} + \frac{\sqrt{33}}{2})$;

b) $p = (x - 1)(x + 1)(x + 4)(x - \sqrt{5})(x + \sqrt{5})$;

c) $p = 8(x + \frac{7}{8})(x - \frac{1}{2} - \frac{i\sqrt{3}}{2})(x - \frac{1}{2} + \frac{i\sqrt{3}}{2})$.