

MATH 111 CHAPTER 5 Review (sections 5.1-5.4)

1. Terms to know: exponential function, natural exponential function, logarithmic function, natural logarithmic function, vertical and horizontal asymptotes, exponential models, growth rate, initial population, radioactive decay, half-life, inverse functions.
2. Be able to find the asymptotes, intercepts, the domain and range and graph exponential and logarithms functions. (sec. 5.1, 5.2)
3. Know and be able to use Laws of Exponents. (sec. 5.1)
4. Be able to solve an exponential equation by factoring, quadratic formula, and using one-to-one property. (sec. 5.1)
5. Know and be able to use properties/laws of logarithms to evaluate and simplify expressions. (sec. 5.2)
6. Be able to rewrite the logarithmic expression so that the result does not contain logarithms of products, quotients, or powers. (sec. 5.2)
7. Be able to use the laws of logarithms to rewrite the given expression as one logarithm. (sec. 5.2)
8. Be able to solve logarithmic equations by factoring, quadratic formula, and using one-to-one property. (sec. 5.3)
9. Be able to find an inverse function for the given exponential or/and logarithmic function. (sec. 5.2)
10. Be able to find an equation of the exponential and/or logarithmic function that satisfies the given conditions: passing through the point, has a specific asymptotes, etc. (sec. 5.1, 5.2)
11. Be able to solve logarithmic and exponential models. (sec. 5.4)
12. Any handouts given in class, any class discussions.

**Partial Review Exercises**

1. Evaluate

a)  $\log_4 \frac{1}{64}$ ; b)  $\log_3(-1)$ ; c)  $5^{\log_5 4}$ ; d)  $e^{3\ln 2}$ ; e)  $\log_{11} \sqrt{11}$ . SLO (Student Learning Outcome) 5

2. Solve the given equations: SLO 4, 8

a)  $5^{2x+1} = 6^{x-2}$ ; b)  $\log x = 1 - \log(x-3)$ ; c)  $2\ln(x+3) - \ln(x+1) = 3\ln 2$ ; d)  $e^{3x+1} = 2^{x+7}$ ; e)  $e^{5-3x} = 10$ ;  
f)  $\log_3(\log_2 x) = 2$ ; g)  $(2^x)^2 - 5 \cdot (2^x) + 4 = 0$ ; h)  $\ln(x+7) + \ln(x-8) - \ln x = \ln 9$ ; i)  $10^{-4x} = \frac{1}{1000}$ ;  
j)  $2^x = 8^{x^2}$ ; k)  $2^{2(x-2)} = 7^2$ ; l)  $(4^x)^2 - 2(4^x) - 1 = 0$ ; m)  $\log_6(2-x) + \log_6(7-x) = 2$ .

3. Find the horizontal asymptote and the y-intercept of the graph: SLO 2

a)  $f(x) = -5 + e^{x-7}$ ; b)  $f(x) = 3 + 5^{-x}$ .

4. Find the exact value of

a)  $\log_2 6 - \log_2 15 + \log_2 20$ ; b)  $\log_5 80 - \log_5 50 - \log_5 40$ ; c)  $\frac{1}{2}\log_2 9 - \frac{1}{3}\log_2 343 + 13\log_2 1$ . SLO 5, 7

5. A culture starts with 10,000 bacteria, and the number doubles every 40 minutes. a) Find a formula for the number of bacteria at time t in min.

b) After how many minutes will there be 50,000 bacteria? SLO 11

6. Polonium-210 ( $^{210}\text{Po}$ ) has a half-life of 140 days. Suppose a sample of this substance has a mass of 300 mg. SLO 11

- a) Find a formula for the amount of the sample remaining after  $t$  days.  
 b) Find the mass remaining after 1 year.

7. Find the inverse function **SLO 9**

a)  $f(x) = \ln(3x + 2)$ ; b)  $g(x) = e^{-x-5} + 1$ ; c)  $f(x) = 7 + 2^x$ ; d)  $f(x) = 1 + \ln(x - 7)$ .

8. Graph the function and state the equation of the asymptote **SLO 2**

a)  $y = -e^{x+2} - 4$ ; b)  $y = 5^{-(x-3)} + 2$ ; c)  $y = \ln(x + 1) - 2$ .

9. Use the properties of logarithms to simplify the expression so that the result does not contain logarithms of products, quotients, or powers. **SLO 6**

a)  $\log(x^3(x+1)^{1/2})$ ; b)  $\ln\left(\frac{s^3\sqrt{t}}{(t^2+1)^4}\right)$ ; c)  $\ln\sqrt{\frac{(2x+1)^3(3x+2)}{(8x+3)^5}}$ .

10. The half-life of strontium-90 is 25 years. If a sample has a mass of 24 grams, find the mass remaining after 40 years. **SLO 11**

11. Initially 200 milligrams of a radioactive substance was present. After 5 hours the mass had decreased by 3%. Find an exponential model for the amount remaining of the decaying substance after  $t$  hours. **SLO 11**

12. Simplify and write as a single logarithm: **SLO 7**

a)  $\ln\left(\frac{x}{y}\right) - 3\ln(x^5) - 13\ln(y)$ ; b)  $\frac{1}{2}\ln 36 + 2\ln 4 - \ln 2$ .

13. Find an exponential function  $f(x) = b^x$  such that the graph of  $f$  passes through the point  $(-6, e^{42})$ . **SLO 10**

14. Find an exponential function  $f(x) = b^x$  such that the graph of  $f$  passes through the point  $(2, 9)$ . **SLO 10**

15. Find a logarithmic function  $f(x) = \log_b x$  such that the graph of  $f$  passes through the point  $(25, 2)$ . **SLO 10**

16. Find the domain of the function: a)  $f(x) = \ln(7x - 9)$ ; b)  $f(x) = \sqrt{\ln x}$ ; c)  $f(x) = \frac{5}{\ln x}$ . **SLO 2**

17. Simplify: a)  $(2m^4n^5)^2$ ; b)  $\frac{9}{3ab^{-8}}$ . **SLO 3**

18. Find the vertical asymptote and the x-intercept of the graph:

a)  $f(x) = 1 - 2\log_9(x - 3)$ ; b)  $f(x) = 2 + \ln(x + 4)$ . **SLO 2**

**Answers:**

1. a) -3; b) dne; c) 4; d) 8; e)  $\frac{1}{2}$ .

2. a)  $x = -\frac{\ln(180)}{\ln(\frac{25}{6})}$ ; b) 5; c) 1; d)  $x = \frac{7\ln 2 - 1}{3 - \ln 2}$ ; e)  $x = \frac{1}{3}(5 - \ln(10))$ ; f)  $x = 512$ ; g)  $x = 0, x = 2$ ; h)  $x = 14$ ; i)  $\frac{3}{4}$ ;

j) 0,  $\frac{1}{3}$ ; k)  $2 + \frac{\ln(7)}{\ln(2)}$ ; l)  $\log_4(1 + \sqrt{2})$ ; m) -2.

3. a)  $y = -5$ ;  $(0, \frac{1}{e^7} - 5)$ ; b)  $y = 3$ ,  $(0, 4)$ .

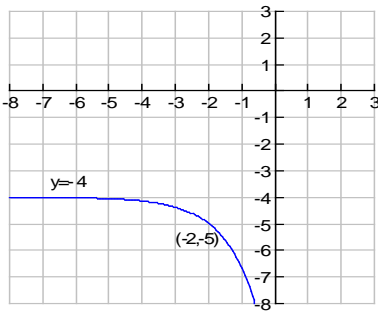
4. a) 3; b) -2, c)  $\log_2(\frac{3}{7})$ .

5. a)  $Q = 10,000e^{\frac{\ln 2}{40}t}$ ; b) 92.9.

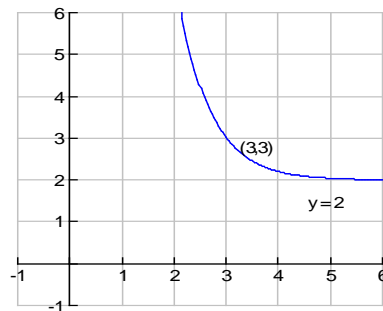
6. a)  $Q = 300e^{\frac{\ln 2}{140}t}$ ; b) 49.256.

7. a)  $f^{-1}(x) = \frac{e^x - 2}{3}$ ; b)  $g^{-1}(x) = \ln(x-1) + 5$ ; c)  $f^{-1} = \log_2(x-7)$ ; d)  $f^{-1}(x) = e^{x-1} + 7$ .

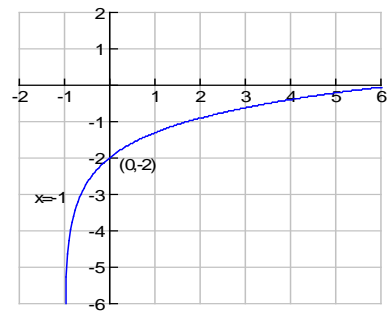
8. a)



b)



c)



9. a)  $3 \log x + \frac{1}{2} \log(x+1)$ ; b)  $3 \ln s + \frac{1}{2} \ln t - 4 \ln(t^2 + 1)$ , c)  $\frac{3}{2} \ln(2x+1) + \frac{1}{2} \ln(3x+2) - \frac{5}{2} \ln(8x+3)$ .

10. 7.9 mg

11.  $A(t) = 200e^{\frac{\ln(194/200)}{5}t}$ .

12. a)  $-14 \ln(xy)$ ; b)  $\ln(48)$ .

13.  $y = e^{-7x}$ .

14.  $y = 3^x$ .

15.  $f(x) = \log_5 x$ .

16. a)  $(\frac{9}{7}, \infty)$ ; b)  $[1, \infty)$ ; c)  $(0, 1) \cup (1, \infty)$ .

17. a)  $4m^8 n^{10}$ ; b)  $\frac{3b^8}{a}$ .

18. a) VA  $x = 3$ ;  $(6, 0)$ ; b) VA  $X = -4$ ,  $(\frac{1}{e^2} - 4, 0)$ .