

Math 101 Review on Exponential and Logarithmic Functions

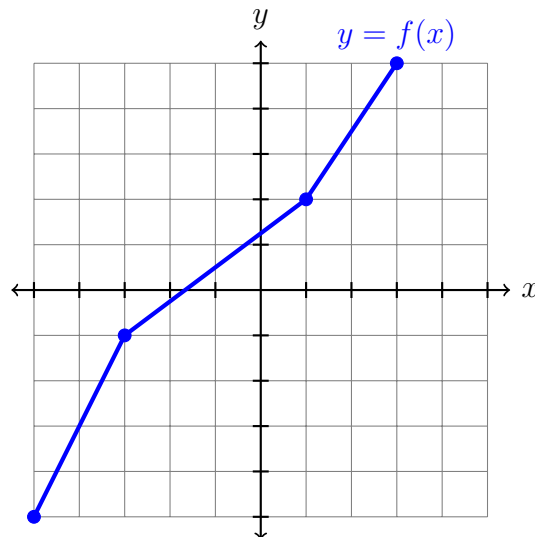
## Learning Goals

- (EX-1) Composite functions  
I can combine functions using composition. I can find the domain of a composite function.
- (EX-2) Inverse functions  
I can determine whether a function is one-to-one. I can obtain the graph of the inverse function from the graph of the function. I can determine the inverse of a function defined by a map or a set of ordered pairs. I can find the inverse of a function defined by an equation.
- (EX-3) Exponential functions  
I can evaluate exponential functions. I can define the number  $e$  and approximate it to five decimal places. I can graph exponential functions. I can solve basic exponential equations.
- (EX-4) Logarithmic functions  
I can change an exponential equation to a logarithmic equation. I can change a logarithmic equation to an exponential equation. I can evaluate logarithmic expressions without using a calculator. I can determine the domain of a logarithmic function. I can graph logarithmic functions. I can solve basic logarithmic equations.

## Review Problems

1. For each pair of functions, find  $H(x) = (f \circ g)(x)$  and simplify. Determine the domain of  $H(x)$ .
 

(a) $f(x) = \sqrt{x+5}$ , $g(x) = 4x - 1$	(b) $f(x) = 4x - 1$ , $g(x) = \sqrt{x+5}$
(c) $f(x) = 2x^2 - 3x$ , $g(x) = x + 9$	(d) $f(x) = x + 9$ , $g(x) = 2x^2 - 3x$
(e) $f(x) = \frac{-3}{x}$ , $g(x) = \frac{x}{x-2}$	(f) $f(x) = \frac{x}{x-2}$ , $g(x) = \frac{-3}{x}$
2. Use the graph of the one-to-one function  $f(x)$  shown below to graph its inverse function  $f^{-1}(x)$ .



3. Find the inverse  $g^{-1}(x)$  for the function  $g(x)$ . State the domain and range of  $g$  and  $g^{-1}$ .

$$g(x) = \frac{x+2}{1-x}$$

4. Find the inverse  $f^{-1}(x)$  for each function  $f(x)$ .

(a)  $f(x) = -2x + 5$

(b)  $f(x) = x^3 - 7$

(c)  $f(x) = \sqrt[3]{x-4}$

(d)  $f(x) = x^2 - 3$  for  $x \geq 0$

(e)  $f(x) = \sqrt{x+5}$

(f)  $f(x) = \{(-4, 13), (-1, 7), (0, 4), (5, 5), (8, 2)\}$

5. Fill in the table of values for  $f(x) = 3^x$  and graph the function.

$x$	$f(x) = 3^x$
-1	
0	
1	
2	
3	

6. Fill in the table of values for  $f(x) = \log_3(x)$  and graph the function.

$x$	$f(x)$
	-1
1	
	1
9	

7. Fill in the table of values for  $f(x) = \left(\frac{1}{5}\right)^x$  and graph the function.

$x$	$f(x) = \left(\frac{1}{5}\right)^x$
-2	
-1	
0	
1	
2	

8. Determine whether the given function is linear, exponential, or neither.

(a) 

$x$	$f(x)$
-1	1
0	5
1	9
2	13
3	17

(b) 

$x$	$f(x)$
-1	1
0	$\frac{1}{5}$
1	$\frac{1}{25}$
2	$\frac{1}{125}$
3	$\frac{1}{625}$

(c) 

$x$	$f(x)$
-1	2
0	5
1	7
2	12
3	19

9. Find the exact value of each logarithm without using a calculator.

(a)  $\ln 1$

(b)  $\log_{20}(20)$

(c)  $\log_5(0.2)$

(d)  $\log_3(27)$

(e)  $\log_{\sqrt{3}}(9)$

(f)  $\log_{1/2}(16)$

(g)  $\ln e^\pi$

(h)  $\ln \sqrt{e}$

10. Convert  $\log_5(K) = a$  to exponential form.

11. Convert  $e^{2x} = R$  to logarithmic form.

12. Solve each equation for  $x$ .

(a)  $5^x = 125$

(b)  $3^{x^2} = 9$

(c)  $8^{-x+14} = 16^x$

(d)  $e^{3x+8} = e^x$

(e)  $\log_3(4x - 7) = 2$

(f)  $6 + \log_3(-4x) = 9$

(g)  $\log_8 x = \frac{1}{3}$

(h)  $3 \ln(x - 7) = 15$

(i)  $16 = 8^{x+1}$

(j)  $e^{2x} = 5$

## Answers

1. (a)  $H(x) = 2\sqrt{x+1}$ , Domain  $x \geq -1$

(b)  $H(x) = 4\sqrt{x+5} - 1$ , Domain  $x \geq -5$

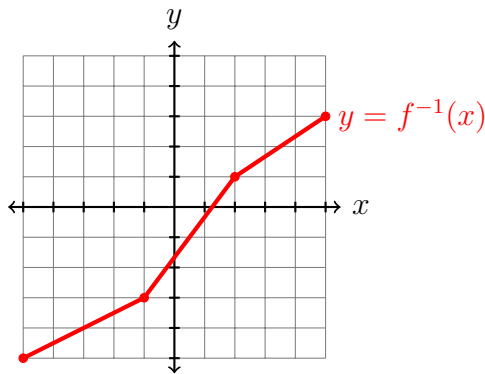
(c)  $H(x) = 2x^2 + 33x + 135$ , Domain  $(-\infty, \infty)$

(d)  $H(x) = 2x^2 - 3x + 9$ , Domain  $(-\infty, \infty)$

(e)  $H(x) = 3x - 6$ , Domain  $x \neq 2$

(f)  $H(x) = \frac{x}{3-2x}$ , Domain  $x \neq 0, x \neq \frac{3}{2}$

2. Graph:



3.  $g^{-1}(x) = \frac{x-2}{x+1}$ , Domain  $g$  is  $x \neq 1$ ; range  $y \neq -1$ . Domain  $g^{-1}$  is  $x \neq -1$ ; range  $y \neq 1$

4. (a)  $f^{-1}(x) = -\frac{1}{2}x + \frac{5}{2}$

(b)  $f^{-1}(x) = \sqrt[3]{x+7}$

(c)  $f^{-1}(x) = (x+4)^3$

(d)  $f^{-1}(x) = \sqrt{x+3}$

(e)  $f^{-1}(x) = x^2 - 5$  for  $x \geq 0$

(f)  $f^{-1}(x) = \{(13, -4), (7, -1), (4, 0), (5, 5), (2, 8)\}$

5. Graph given below.

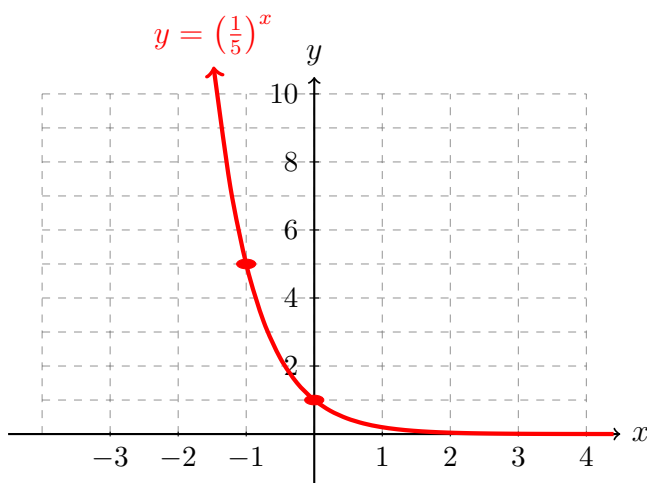
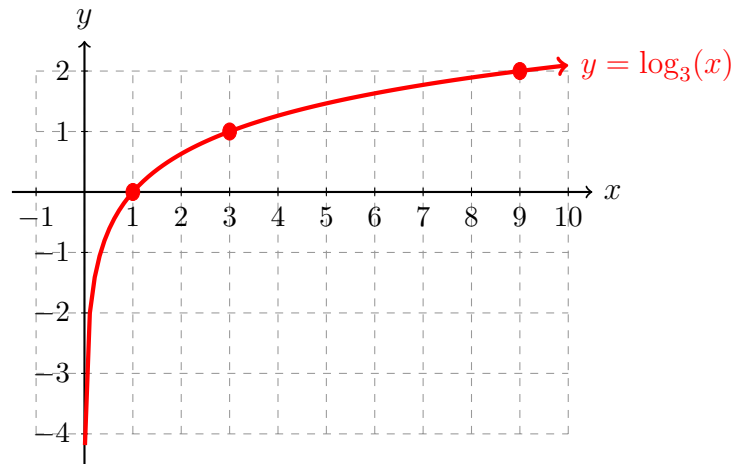
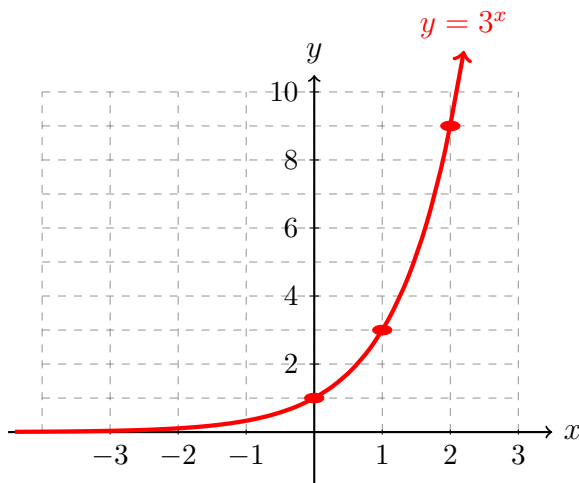
$x$	$f(x) = 3^x$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

6. Graph given below.

$x$	$f(x)$
$\frac{1}{3}$	-1
1	0
3	1
9	2

7. Graph given below.

$x$	$f(x) = \left(\frac{1}{5}\right)^x$
-2	25
-1	5
0	1
1	$\frac{1}{5}$
2	$\frac{1}{25}$



8. (a) Linear; (b) Exponential; (c) Neither

9. (a) 0 (b) 1 (c) -1 (d) 3 (e) 4 (f) -4 (g)  $\pi$  (h)  $1/2$

10.  $5^a = K$

11.  $\ln(R) = 2x$

12. (a)  $x = 3$  (b)  $x = \pm\sqrt{2}$  (c)  $x = 6$  (d)  $x = -4$   
 (e)  $x = 4$  (f)  $x = -27/4$  (g)  $x = 2$  (h)  $x = 7 + e^5$   
 (i)  $x = 1/3$  (j)  $x = \frac{\ln(5)}{2}$