Math 101 Review on Polynomials & Rational Functions

Learning Goals

□ (PY-1) Properties of Polynomial functions
I can identify polynomial functions and their degree. I can graph polynomial functions using transformations. I can analyze the graph of polynomial functions. I can identify the real zeros of polynomial functions and their multiplicity.

□ (PY-2) Properties of Rational functions
I can find the domain of a rational function. I can find the vertical asymptotes of a rational function. I can find the horizontal asymptotes of a rational function. I can find the oblique asymptotes of a rational function. I can find any intercepts appearing on the graph of a rational function.

□ (PY-3) Polynomial and Rational inequalities
I can produce a completely correct Sign Chart without any errors. I can solve polynomial inequalities (including quadratic). I can solve rational inequalities.

Review Problems

1. For each polynomial function, determine the degree, the leading coefficient, and each real zero with its multiplicity.
   - (a) $f(x) = x^2 - 4x + 3$
   - (b) $f(x) = (2x + 10)^2$
   - (c) $f(x) = (x - 2)^5$
   - (d) $f(x) = x^3 - 8$
   - (e) $f(x) = 4(x + 4)(x + 3)^3$
   - (f) $f(x) = -x^2(x^2 - 25)$

2. For each polynomial, determine which of the following descriptions apply:
   - I. The graph rises on the left.
   - II. The graph falls on the left.
   - III. The graph rises on the right.
   - IV. The graph falls on the right.

   - (a) $f(x) = -x^5 - 7x^4 + 6x^3 - 9$
   - (b) $f(x) = -4x^4 + 6x^3 - 2x - 5$
   - (c) $f(x) = 2x(x - 1)^2(x + 3)$
   - (d) $f(x) = -x(x + 3)^2(x - 3)^2$

3. For each rational function below, find the domain, find the equations of any asymptotes (vertical, horizontal, or oblique), and state the location of any intercepts appearing on each graph.
   - (a) $f(x) = \frac{3x}{x^2 + 2}$
   - (b) $f(x) = \frac{x^2 - 9}{x^2 - 1}$
   - (c) $f(x) = \frac{x^2 - x - 6}{x^2 + x - 6}$
   - (d) $f(x) = \frac{2x^3}{x^2 + 1}$
   - (e) $f(x) = \frac{(x + 1)(x + 3)}{x}$

4. Solve the inequalities given below. Write your answers in interval notation. If no solution exists, write “∅”.

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(a) \((x + 3)(x - 1) < 0\) 
(b) \(-3x^2 + x \leq -4\) 
(c) \(-(x - 25)^2 > 0\) 
(d) \(-x^2 + 6x - 9 \leq 0\) 
(e) \((x - 4)^2(x + 3) < 0\) 
(f) \(x^3 + 10x^2 < 0\) 
(g) \(x^3 \leq 9x\) 
(h) \(\frac{x - 3}{x + 1} > 0\) 
(i) \(\frac{(x + 1)^2}{x^2 - 4} \geq 0\) 
(j) \(\frac{x + 2}{x - 4} \geq 1\) 
(k) \(\frac{x + 4}{2x + 4} \geq 1\)

**Answers**

1a. Degree 2, leading coeff. 1, \(x = -1\) mult. 1 and \(x = -3\) mult. 1

1b. Degree 2, leading coeff. 4, \(x = -5\) mult. 2

1c. Degree 5, leading coeff. 1, \(x = 2\) mult. 5

1d. Degree 3, leading coeff. 1, \(x = 2\) mult. 1

1e. Degree 4, leading coeff. 4, \(x = -4\) mult. 1 and \(x = -3\) mult. 3

1f. Degree 4, leading coeff. \(-1\), \(x = 0\) mult. 2, \(x = 5\) mult. 1, \(x = -5\) mult. 1

2. (a) I, IV  (b) II, IV  (c) I, III, (d) I, IV

3a. Domain: \((-\infty, \infty)\). HA: \(y = 0\). x-int: \((0,0)\). y-int: \((0,0)\)

3b. Domain: \(x \neq \pm 1\). VA: \(x = -1, x = 1\). HA: \(y = 1\). x-int: \((-2,0), (2,0)\). y-int: \((0,9)\)

3c. Domain: \(x \neq -3, x \neq 2\). VA: \(x = -3, x = 2\). HA: \(y = 1\). x-int: \((-2,0), (3,0)\). y-int: \((0,1)\)

3d. Domain: \((-\infty, \infty)\). OA: \(y = 2x\). x-int: \((0,0)\). y-int: \((0,0)\).

3e. Domain: \((-\infty, 0) \cup (0, \infty)\). VA: \(x = 0\). OA: \(y = x + 4\). x-int: \((-1,0), (-3,0)\). y-int: None.

4.

(a) \((-3,1)\)  
(b) \((-\infty, -1] \cup \left[\frac{4}{3}, \infty\right)\)  
(c) \(\emptyset\)  
(d) \((-\infty, \infty)\)  
(e) \((-\infty, -3)\)  
(f) \((-\infty, -10)\)  
(g) \((-\infty, -3] \cup [0,3]\)  
(h) \((-\infty, -1) \cup (3, \infty)\)  
(i) \((-\infty, -2) \cup (2, \infty)\)  
(j) \((4, \infty)\)  
(k) \([-8, -2)\)