Math 101 Review on Functions

Learning Goals

□ (FN-1) Functions, domains, and difference quotients
I can determine whether a relation represents a function and find the value of a function. I can find and simplify the difference quotient of a function. I can find the domain of a function defined by an equation. I can combine functions using addition, subtraction, multiplication, and division. I can identify the graph of a function and I can use a function’s graph to obtain information about the function.

□ (FN-2) Linear functions and models
I can graph a linear function. I can determine whether a linear function is increasing, decreasing or constant. I can determine the average rate of change of a linear function and use it to identify linear functions. I can build linear models from verbal descriptions and use the models to establish conclusions.

□ (FN-3) Quadratic functions and models
Given a quadratic equation, I can identify the vertex and the axis of symmetry on its graph. I can graph a quadratic function using its equation. I can graph a quadratic function using its vertex and one other point. I can find and identify x-intercepts on the graph of a quadratic function. I can use an equation or a graph to find the minimum or maximum value of a quadratic function.

Review Problems

1. Compute and simplify the difference quotient \( \frac{f(x + h) - f(x)}{h} \) for each function listed below.
   
   (a) \( f(x) = -3x + 5 \)  \hspace{1cm} (b) \( f(x) = 7 + \frac{2}{3}x \)  \hspace{1cm} (c) \( f(x) = x^2 - 3x \)
   
   (d) \( f(x) = -2x^2 + 3x - 1 \)  \hspace{1cm} (e) \( f(x) = x^3 - 2x + 9 \)  \hspace{1cm} (f) \( f(x) = \frac{1}{x} \)
   
   (g) \( f(x) = \frac{3}{x + 1} \)

2. Find the domain of each function given below. Write your answer as an interval or a union of intervals.
   
   (a) \( f(x) = \sqrt{5 - 2x} \)  \hspace{1cm} (b) \( f(x) = \frac{14x}{2x - 5} \)  \hspace{1cm} (c) \( f(x) = \frac{\sqrt{x - 7}}{x - 10} \)

3. Fill in the table below using this function rule: \( f(x) = \sqrt{8 - x} \). Simplify your answers as much as possible.

   \[
   \begin{array}{c|c}
   x & f(x) \\
   \hline
   -41 & \hspace{1cm} \\
   -1 & \\
   8 & \\
   9 & \\
   \end{array}
   \]
4. Suppose that the relation $T$ is defined as follows.

$$T = \{(6, 6), (8, a), (a, 8), (0, b)\}$$

Give the domain and range of $T$. Write your answer using set notation.

5. The functions $f$, $g$, and $h$ are defined as follows:

$$f(x) = 3 + \sqrt{x + 8}, \quad g(x) = \frac{3 + x^2}{x + 4}, \quad h(x) = \left|\frac{-1}{4}x - 13\right|$$

Find $f(-4)$, $g(4)$, and $h(16)$. Simplify your answers.

6. Suppose that the functions $f$ and $g$ are defined as follows:

$$f(x) = 3x + 4 \quad g(x) = 5x^2 - 6x + 3$$

(a) Find the function $f + g$ and simplify.
(b) Find the function $g - f$ and simplify.
(c) Find the function $f \cdot g$ and simplify.
(d) Find the function $\frac{f}{g}$.

7. Five years after purchase, the owner of a bookstore estimates the value of the business as $60,000. Eight years after purchase, the owner estimates the value of the business has depreciated to $42,000. Find a linear function $B(t)$ that models the value of the business as a function of years $t$ since purchase. How much will the business be worth after 11 years?

8. Sketch the graph of a linear function satisfying each description and state the equation of each function you’ve drawn.

(a) $m < 0, b < 0$ \quad (b) $m < 0, b > 0$ \quad (c) $m < 0, b = 0$ \quad (d) $m > 0, b > 0$
(e) $m > 0, b < 0$ \quad (f) $m > 0, b = 0$ \quad (g) $m = 0, b < 0$ \quad (h) $m = 0, b > 0$

9. For the quadratic function $f(x) = -2x^2 - 16x - 27$,

(a) Write the quadratic function in the form $f(x) = a(x - h)^2 + k$.
(b) Identify the vertex and axis of symmetry.
(c) Graph the function.

10. Graph the function $f(x) = x^2 - 4x - 1$. Label the vertex and at least two other points on the parabola.

11. Use the graph of the parabola to answer the questions below:
12. Find the equation of the quadratic function $g$ whose vertex is found at $(-3, -4)$ and whose graph goes through the point $(-6, -13)$.

13. Determine the maximum or minimum value of $g(x) = -2x^2 + 4x - 5$.

14. Determine all $x$-intercepts found on the graph of $f(x) = -4x^2 + 9$.

15. The profit for a company selling coffee tables is given by the function $P(x) = -1.6x^2 + 240x - 375$, where $P(x)$ is the profit in dollars and $x$ is the number of tables sold.

(a) Find the $y$-intercept and explain what it means in this context.

(b) How many tables should be sold to maximize profit?

16. A projectile is thrown upward with an initial velocity of 176 feet per second. After $t$ seconds, the height of the projectile is given by $h(t) = -16t^2 + 176t$, where $h$ is in feet.

(a) Find the projectile’s height after 2 seconds.

(b) What is the projectile’s maximum height? What is the value of $t$ at this height?

(c) After how many seconds will the projectile hit the ground?

Answers

1. (a) $-3$, (b) $\frac{2}{3}$, (c) $2x - 3 + h$, (d) $-4x + 3 - 2h$, (e) $3x^2 + 3xh + h^2 - 2$, (f) $\frac{-1}{(x+h)^2}$, (g) $\frac{-3}{(x+1)(x+h+1)}$

2. (a) $(-\infty, \frac{5}{2})$, (b) $(-\infty, \frac{5}{2} \cup (\frac{5}{2}, \infty)$, (c) $[7, 10) \cup (10, \infty)$

3. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
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</thead>
<tbody>
<tr>
<td>$-41$</td>
<td>$7$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$3$</td>
</tr>
<tr>
<td>$8$</td>
<td>$0$</td>
</tr>
<tr>
<td>$9$</td>
<td>$i$</td>
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</tbody>
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4. Domain $\{6, 8, a, 0\}$, Range $\{6, a, 8, b\}$

5. $f(-4) = 5$, $g(4) = 19/8$, $h(16) = 17$. 
6. (a) \((f+g)(x) = 5x^2-3x+7\), (b) \((g-f)(x) = 5x^2-9x-1\), (c) \((f \cdot g)(x) = 15x^3+2x^2-15x+12\), (d) \(\left(\frac{f}{g}\right)(x) = \frac{3x+4}{5x^2-6x+3}\).

7. \(B(t) = -6000t + 90000\), \(B(11) = 24,000\)

8. Answers will vary.

9. (a) \(f(x) = -2(x + 4)^2 + 5\), (b) Vertex: \((-4,5)\), axis of symmetry: \(x = -4\),

Problem 9

[Diagram of a parabola with the vertex at \((-4,5)\) and a point at \((-6,-3)\).]

Problem 10

[Diagram of a parabola with points \((2,-5)\), \((0,-1)\), and \((4,-1)\).]

10. Graph given above.

11. (a) upward, (b) \(x = -4\), (c) \((-4,2)\), (d) None, (e) \(y\)-int: \((0,6)\)

12. \(g(x) = -1(x + 3)^2 - 4\)

13. Max value of \(-3\) at \(x = 1\); no min.

14. \((\pm \frac{3}{2}, 0)\)

15. (a) \((0,-375)\); when no tables are sold, the company will lose \$375. (b) 75

16. (a) 288 feet, (b) 484 feet at 5.5 seconds, (c) 11 seconds