Math 101 Review on Functions

Learning Goals

☐ (FN-1) Functions, domains, and difference quotients
I can determine whether a relation represents a function and find the value of a function. I can find and simplify the difference quotient of a function. I can find the domain of a function defined by an equation. I can combine functions using addition, subtraction, multiplication, and division. I can identify the graph of a function and I can use a function's graph to obtain information about the function.

☐ (FN-2) Linear functions and models
I can graph a linear function. I can determine whether a linear function is increasing, decreasing or constant. I can determine the average rate of change of a linear function and use it to identify linear functions. I can build linear models from verbal descriptions and use the models to establish conclusions.

☐ (FN-3) Quadratic functions and models
Given a quadratic equation, I can identify the vertex and the axis of symmetry on its graph. I can graph a quadratic function using its equation. I can graph a quadratic function using its vertex and one other point. I can find and identify x-intercepts on the graph of a quadratic function. I can use an equation or a graph to find the minimum or maximum value of a quadratic function.

Review Problems

1. Compute and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for each function listed below.

   (a) $f(x) = -3x + 5$ (b) $f(x) = 7 + \frac{2}{3}x$ (c) $f(x) = x^2 - 3x$

   (d) $f(x) = -2x^2 + 3x - 1$ (e) $f(x) = x^3 - 2x + 9$ (f) $f(x) = \dfrac{1}{x}$

   (g) $f(x) = \dfrac{3}{x+1}$

2. Find the domain of each function given below. Write your answer as an interval or a union of intervals.

   (a) $f(x) = \sqrt{5-2x}$ (b) $f(x) = \dfrac{14x}{2x-5}$ (c) $f(x) = \dfrac{\sqrt{x-7}}{x-10}$

3. Fill in the table below using this function rule: $f(x) = \sqrt{8-x}$. Simplify your answers as much as possible.

x	$f(x)$
-41	
-1	
8	
9	

4. Suppose that the relation T is defined as follows.

$$T = \{(6,6), (8,a), (a,8), (0,b)\}$$

Give the domain and range of T. Write your answer using set notation.

5. The functions f, g, and h are defined as follows:

$$f(x) = 3 + \sqrt{x+8}, \qquad g(x) = \frac{3+x^2}{x+4}, \qquad h(x) = \left|\frac{-1}{4}x - 13\right|$$

Find $f(-4)$, $g(4)$, and $h(16)$. Simplify your answers.

6. Suppose that the functions f and g are defined as follows:

$$f(x) = 3x + 4$$
$$g(x) = 5x^2 - 6x + 3$$

(a) Find the function $f + g$ and simplify.

(b) Find the function $g - f$ and simplify.

(c) Find the function $f \cdot g$ and simplify.

(d) Find the function $\frac{f}{g}$.

7. Five years after purchase, the owner of a bookstore estimates the value of the business as \$60,000. Eight years after purchase, the owner estimates the value of the business has depreciated to \$42,000. Find a linear function $B(t)$ that models the value of the business as a function of years t since purchase. How much will the business be worth after 11 years?

8. Sketch the graph of a linear function satisfying each description and state the equation of each function you've drawn.

(a) $m < 0, b < 0$ (b) $m < 0, b > 0$ (c) $m < 0, b = 0$ (d) $m > 0, b > 0$

(e) $m > 0, b < 0$ (f) $m > 0, b = 0$ (g) $m = 0, b < 0$ (h) $m = 0, b > 0$

9. For the quadratic function $f(x) = -2x^2 - 16x - 27$,

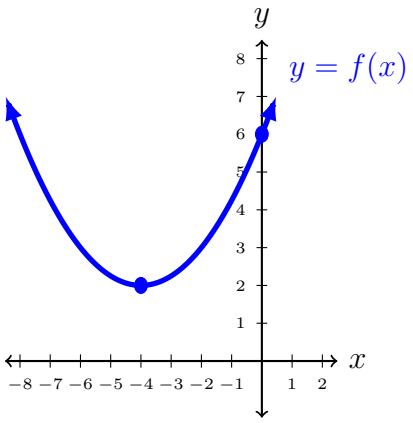
(a) Write the quadratic function in the form $f(x) = a(x-h)^2 + k$.

(b) Identify the vertex and axis of symmetry.

(c) Graph the function.

10. Graph the function $f(x) = x^2 - 4x - 1$. Label the vertex and at least two other points on the parabola.

11. Use the graph of the parabola to answer the questions below:



(a) Does the parabola open upward or downward?

(b) Find the equation of the axis of symmetry.

(c) Find the coordinates of the vertex.

(d) Find the x-intercept(s). If none, write "none."

(e) Find the y-intercept(s). If none, write "none."

12. Find the equation of the quadratic function g whose vertex is found at $(-3, -4)$ and whose graph goes through the point $(-6, -13)$.

13. Determine the maximum or minimum value of $g(x) = -2x^2 + 4x - 5$.

14. Determine all x-intercepts found on the graph of $f(x) = -4x^2 + 9$.

15. The profit for a company selling coffee tables is given by the function $P(x) = -1.6x^2 + 240x - 375$, where $P(x)$ is the profit in dollars and x is the number of tables sold.

    (a) Find the y-intercept and explain what it means in this context.

    (b) How many tables should be sold to maximize profit?

16. A projectile is thrown upward with an initial velocity of 176 feet per second. After t seconds, the height of the projectile is given by $h(t) = -16t^2 + 176t$, where h is in feet.

    (a) Find the projectile's height after 2 seconds.

    (b) What is the projectile's maximum height? What is the value of t at this height?

    (c) After how many seconds will the projectile hit the ground?

Answers

1. (a) -3, (b) $\frac{2}{3}$, (c) $2x - 3 + h$, (d) $-4x + 3 - 2h$, (e) $3x^2 + 3xh + h^2 - 2$, (f) $\frac{-1}{(x+h)x}$, (g) $\frac{-3}{(x+1)(x+h+1)}$

2. (a) $\left(-\infty, \frac{5}{2}\right]$, (b) $\left(-\infty, \frac{5}{2}\right)$ cup $\left(\frac{5}{2}, \infty\right)$, (c) $[7, 10) \cup (10, \infty)$

3.

x	$f(x)$
-41	7
-1	3
8	0
9	i

4. Domain $\{6, 8, a, 0\}$, Range $\{6, a, 8, b\}$

5. $f(-4) = 5$, $g(4) = 19/8$, $h(16) = 17$.

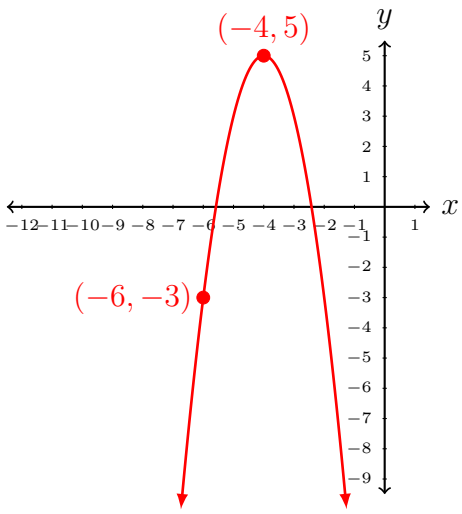
6. (a) $(f+g)(x) = 5x^2-3x+7$, (b) $(g-f)(x) = 5x^2-9x-1$, (c) $(f\cdot g)(x) = 15x^3+2x^2-15x+12$, (d) $\left(\frac{f}{g}\right)(x) = \frac{3x+4}{5x^2-6x+3}$

7. $B(t) = -6000t + 90000$, $B(11) = \$24,000$

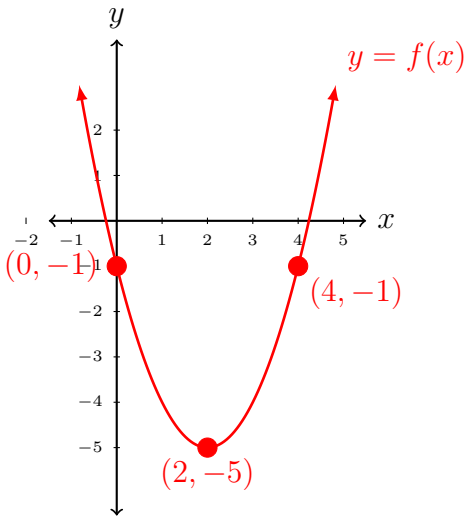
8. Answers will vary.

9. (a) $f(x) = -2(x+4)^2 + 5$, (b) Vertex: $(-4, 5)$, axis of symmetry: $x = -4$,

Problem 9



Problem 10



10. Graph given above.

11. (a) upward, (b) $x = -4$, (c) $(-4, 2)$, (d) None, (e) y-int: $(0, 6)$

12. $g(x) = -1(x+3)^2 - 4$

13. Max value of -3 at $x = 1$; no min.

14. $\left(\pm\frac{3}{2}, 0\right)$

15. (a) $(0, -375)$; when no tables are sold, the company will lose \$375. (b) 75

16. (a) 288 feet, (b) 484 feet at 5.5 seconds, (c) 11 seconds