Math 101 Review on Graphs and Geometry

Learning Goals

☐ (GR-1) Distances, midpoints, and geometric shapes
I know the distance formula and can apply it when appropriate. I know the midpoint formula and can apply it when appropriate. I know and can apply geometry formulas related to squares, triangles, circles, boxes, spheres, and right circular cylinders. I can use the Pythagorean Theorem and its converse.

☐ (GR-2) Graphs of equations, intercepts and symmetry
I can test an equation for symmetry with respect to the $x$-axis, the $y$-axis, and the origin. I can identify symmetry from a graph or complete a graph so that it has a given type of symmetry. I can quickly and accurately graph each of the following basic equations, describing any intercepts or symmetry: $y = x^2, y = x^3, y = x, y = \frac{1}{x}, y = \sqrt{x}, x = y^2, y = |x|$. I can graph functions of the form $f(x) = k, f(x) = x, f(x) = x^2, f(x) = \sqrt{x}, f(x) = \sqrt{x}, f(x) = \frac{1}{x}$, and $f(x) = |x|$. I can graph piecewise-defined functions and I can determine the equation given the graph of a piecewise-defined function.

☐ (GR-3) Graphs of lines and systems of lines
I can find the equation of a line given its slope and a point. I can find the equation of a line given its slope and its $y$-intercept. I can find the equation of a line given two points on the line. I can find equations of parallel lines and perpendicular lines. I can write the equation of a line in slope-intercept form, standard form, and point-slope form. I can graph a line. I can identify the slope and $y$-intercept of a line from its equation or graph. I can solve systems of linear equations in two variables by substitution and I can solve systems of linear equations in two variables by elimination. I can identify inconsistent systems of equations in two variables and I can express the solution of a system of dependent equations containing two variables.

☐ (GR-4) Circles
I can convert between standard form and expanded form for the equation of a circle. Given its properties, I can graph a circle and find its equation. I can find $x$-intercepts and $y$-intercepts found on the graph of a circle.

Review Problems

1. For the following right triangle, find the side of length $x$. Simplify your answer.

![Right Triangle Diagram]

2. Find the exact area of a circle of radius 5 feet.

3. Graph the line $x - y = -4$.

4. Graph the line with slope $-1$ passing through the point $(-1, -2)$. 

5. Graph the line with slope \( m = -3 \) and \( y \)-intercept \( b = -1 \).

6. Find an equation for the line going through \((-1, -1)\) and \((3, 1)\).

7. A line passes through the point \((2, -4)\) and has a slope of 4. Write an equation in point-slope form for this line.

8. Calculate the distance between the points \( F = (1, -6) \) and \( J = (7, -1) \) in the coordinate plane. Give an exact answer.

9. Find the slope, the \( x \)-intercept(s), and the \( y \)-intercept(s) of the line \( 5x + 3y = -12 \). Write the line’s equation in slope-intercept form.

10. Find the midpoint \( M \) of the line segment joining the points \( A = (-7, 6) \) and \( B = (-1, -4) \).

11. Consider the line \( 4x - 7y = -8 \).
   
   (a) Find the equation of the line that is perpendicular to this line and passes through the point \((4, -4)\).
   
   (b) Find the equation of the line that is parallel to this line and passes through the point \((4, -4)\).

12. Find an equation of the circle that has center \((-2, -6)\) and passes through \((2, 2)\).

13. Find the slope and the \( y \)-intercept of the line \( 8x - 4y = 7 \). Simplify your answers.

14. Write equations for the vertical and horizontal lines passing through the point \((-4, 3)\).

15. For each pair of points, find the slope of the line passing through them. If the slope is undefined, write “undefined.” Simplify your answers.
   
   (a) \((-7, 6)\) and \((-9, 8)\)
   
   (b) \((-5, 2)\) and \((6, 2)\)
   
   (c) \((-5, 4)\) and \((-5, -5)\)

16. The equation of a circle is \( x^2 + y^2 - 8x + 2y = -13 \). Identify its radius and center.

17. Find an equation of the circle whose diameter has endpoints \((-6, -4)\) and \((2, -1)\).

18. For each ordered pair below, determine whether it is a solution to the system of equations.

\[
\begin{align*}
\begin{cases}
  y = 2x + 8 \\
  4x - 2y = -16 
\end{cases}
\end{align*}
\]

\[
\begin{array}{ccc}
(x, y) & \text{Yes} & \text{No} \\
\hline
(0, -7) & & \\
(5, -3) & & \\
(-3, 2) & & \\
(5, 18) & & \\
\end{array}
\]

19. Solve each system.

(a) \[
\begin{cases}
  3x + 2y = -6 \\
  x - 3y = -13 
\end{cases}
\]

(b) \[
\begin{cases}
  -9x - 5y = 4 \\
  -4x - 9y = -5 
\end{cases}
\]

(c) \[
\begin{cases}
  x - 4y = -8 \\
  4y = x + 8 
\end{cases}
\]

(d) \[
\begin{cases}
  -x + 2y = -4 \\
  x - 2y = -4 
\end{cases}
\]
20. Identify the center and radius of the circle $x^2 + y^2 - 4x + 6y = -3$. Also, find the $x$-intercept(s) and the $y$-intercept(s) found on its graph, if any.

21. Determine whether the equation has a graph that is symmetric with respect to the $y$-axis, the $x$-axis, the origin, or none of these.

   (a) $y = 3x^2 + 4$
   (b) $x^2 - y^2 = 4$
   (c) $y = (x - 6)(x - 6)$
   (d) $x = y^2 - 16$

22. Draw a complete graph so that it has the type of symmetry indicated.

   (a) $y$-axis
   (b) $x$-axis
   (c) origin

23. Find $f(0)$, $f(8)$, and $f(11)$, where $f(x)$ is the piecewise-defined function

   
   $$f(x) = \begin{cases} 
   8x + 1, & \text{if } x < 1 \\
   8x, & \text{if } 1 \leq x < 11 \\
   8 - 5x, & \text{if } x \geq 11 
   \end{cases}$$

24. Graph the function $f(x) = \begin{cases} 
   -3, & \text{if } x \geq 1 \\
   -5 - x, & \text{if } x < 1 
   \end{cases}$

25. Graph the function $f(x) = \begin{cases} 
   x^3, & \text{if } x < 1 \\
   -2 + x, & \text{if } x \geq 2 
   \end{cases}$

26. The graph of a piecewise-defined function is given. Write its equation.
Answers

1. \( x = 12 \)

2. Area: \( 25\pi \) square feet

3. \( y = x + 4 \)

4. \( y = -1x - 3 \)

5. \( y = -3x - 1 \)

6. \( y = \frac{1}{2}x - \frac{1}{2} \)

7. \( y + 4 = 4(x - 2) \)

8. \( \sqrt{61} \)

9. x-int: \( \left( \frac{12}{5}, 0 \right) \), y-int: \( (0, 4) \), \( y = -\frac{5}{2}x + 4 \)

10. \( M = (-4, 1) \)

11. (a) \( 7x + 4y = 12 \), (b) \( 4x - 7y = 44 \)

12. \( (x + 2)^2 + (y + 6)^2 = 80 \)

13. slope: \( m = 2 \), int: \( b = -7/4 \)

14. vertical: \( x = -4 \), horizontal: \( y = 3 \)

15. (a) \( m = -1 \), (b) \( m = 0 \), (c) undefined

16. \( r = \sqrt{30}, C = (4, -1) \)

17. No, no, yes, yes

18. (a) \( -4, 3 \) , (b) \( -1, 1 \) , (c) \( (t, \frac{1}{4}t + 2) \) where \( t \) is any number, (d) No solution

19. Center: \( C = (2, -3) \), radius: \( r = \sqrt{10} \). x-int: \( (1, 0), (3, 0) \), y-int: \( (0, -3 \pm \sqrt{6}) \)
20. (a) $y$-axis,  (b) $x$-axis, $y$-axis, origin,  (c) none,  (d) $x$-axis

21. (a) $y$-axis   (b) $x$-axis   (c) origin

Graphs:

22. $f(0) = 1$, $f(8) = 64$, $f(11) = -47$

23. Graph the function $f(x) = \begin{cases} -3, & \text{if } x \geq 1 \\ -5 - x, & \text{if } x < 1 \end{cases}$ Graph the function $f(x) = \begin{cases} x^3, & \text{if } x < 1 \\ -2 + x, & \text{if } x \geq 2 \end{cases}$