College of Charleston
Department of Mathematics
Math 101: College Algebra
Final Exam Review Problems[1]

# Learning Goals

- ☐ (AL-1) Arithmetic of Real and Complex Numbers:
  I can classify numbers as natural, integer, whole, irrational, rational, and real. I can evaluate and simplify arithmetic expressions without the use of a calculator. I can add, subtract, multiply, rationalize, and divide complex numbers and simplify them into standard form $a + bi$.
- ☐ (AL-2) Polynomials:
  I can characterize polynomials as monomials, binomials, or trinomials when applicable. I can write polynomials in standard form, identify their degree, and describe the coefficients. I can perform operations on polynomials, including addition, subtraction, multiplication, and (long) division. I can state and apply the formulas for the squares of binomials and the cubes of binomials. I can state and apply the formulas for the difference of two cubes and the sum of two cubes.
- ☐ (AL-3) Factoring Polynomials:
  I can factor polynomials using a variety of techniques. I can complete the square of a polynomial expression. I can factor polynomials in the form $A^3 + B^3$ or $A^3 - B^3$.
- ☐ (AL-4) Rational Expressions:
  I can perform algebraic operations on rational expressions, including reducing to lowest terms, addition, subtraction, multiplication, and division. I can simplify complex rational expressions.
- ☐ (AL-5) Radicals and Exponents:
  I can use the laws of exponents, including those involving roots. I can simplify expressions involving $n$th roots. I can simplify expressions involving rational exponents. I can rationalize the denominator or numerator of a given rational expression.
- ☐ (EQ-1) Linear Equations and Linear Models
  I can solve linear equations in one variable. I can model situations using linear equations and use linear models to form conclusions.
- ☐ (EQ-2) Quadratic Equations and Quadratic Models
  I can find all real and complex solutions to quadratic equations using a variety of methods, including factoring, completing the square, and using the quadratic formula. I can model situations using quadratic equations and use the model to form conclusions. I can describe the discriminant of a quadratic equation and use it to determine its number of solutions over the real numbers.
- ☐ (EQ-3) Radical Equations
  I can solve radical equations and check for extraneous solutions.
- ☐ (EQ-4) Factorable Equations and Equations in Quadratic Form
  I can use factoring to solve quadratic-like equations. I can solve equations in quadratic form.
- ☐ (EQ-5) Inequalities, Intervals, and Graphs
  I can convert between interval notation, inequality notation, and graphical notation for subsets of real numbers. I can use properties of inequalities to simplify, to solve inequalities, and to combine

[1] Please send any corrections to owensks@cofc.edu

inequalities. I can graph solution sets on the real line. I can model real-world problems using inequalities and form conclusions using the model.

☐ (EQ-6) Absolute Value Equations and Inequalities
I can solve equations involving absolute value. I can solve inequalities involving absolute value.

☐ (GR-1) Distances, midpoints, and geometric shapes
I know the distance formula and can apply it when appropriate. I know the midpoint formula and can apply it when appropriate. I know and can apply geometry formulas related to squares, triangles, circles, boxes, spheres, and right circular cylinders. I can use the Pythagorean Theorem and its converse.

☐ (GR-2) Graphs of equations, intercepts and symmetry
I can test an equation for symmetry with respect to the $x$-axis, the $y$-axis, and the origin. I can identify symmetry from a graph or complete a graph so that it has a given type of symmetry. I can quickly and accurately graph each of the following basic equations, describing any intercepts or symmetry: $y = x^2$, $y = x^3$, $y = x$, $y = \frac{1}{x}$ ,$y = \sqrt{x}$, $x = y^2$,$y = |x|$. I can graph functions of the form $f(x) = k$, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sqrt[3]{x}$, $f(x) = \frac{1}{x}$, and $f(x) = |x|$. I can graph piecewise-defined functions and I can determine the equation given the graph of a piecewise-defined function.

☐ (GR-3) Graphs of lines and systems of lines
I can find the equation of a line given its slope and a point. I can find the equation of a line given its slope and its $y$-intercept. I can find the equation of a line given two points on the line. I can find equations of parallel lines and perpendicular lines. I can write the equation of a line in slope-intercept form, standard form, and point-slope form. I can graph a line. I can identify the slope and $y$-intercept of a line from its equation or graph. I can solve systems of linear equations in two variables by substitution and I can solve systems of linear equations in two variables by elimination. I can identify inconsistent systems of equations in two variables and I can express the solution of a system of dependent equations containing two variables.

☐ (GR-4) Circles
I can convert between standard form and expanded form for the equation of a circle. Given its properties, I can graph a circle and find its equation. I can find $x$-intercepts and $y$-intercepts found on the graph of a circle.

☐ (FN-1) Functions, domains, and difference quotients
I can determine whether a relation represents a function and find the value of a function. I can find and simplify the difference quotient of a function. I can find the domain of a function defined by an equation. I can combine functions using addition, subtraction, multiplication, and division. I can identify the graph of a function and I can use a function's graph to obtain information about the function.

☐ (FN-2) Linear functions and models
I can graph a linear function. I can determine whether a linear function is increasing, decreasing or constant. I can determine the average rate of change of a linear function and use it to identify linear functions. I can build linear models from verbal descriptions and use the models to establish conclusions.

☐ (FN-3) Quadratic functions and models
Given a quadratic equation, I can identify the vertex and the axis of symmetry on its graph. I can graph a quadratic function using its equation. I can graph a quadratic function using its vertex

and one other point. I can find and identify $x$-intercepts on the graph of a quadratic function. I can use an equation or a graph to find the minimum or maximum value of a quadratic function.

☐ (PY-1) Properties of Polynomial functions
I can identify polynomial functions and their degree. I can graph polynomial functions using transformations. I can analyze the graph of polynomial functions. I can identify the real zeros of polynomial functions and their multiplicity.

☐ (PY-2) Properties of Rational functions
I can find the domain of a rational function. I can find the vertical asymptotes of a rational function. I can find the horizontal asymptotes of a rational function. I can find the oblique asymptotes of a rational function. I can find any intercepts appearing on the graph of a rational function.

☐ (PY-3) Polynomial and Rational inequalities
I can produce a completely correct Sign Chart without any errors. I can solve polynomial inequalities (including quadratic). I can solve rational inequalities.

☐ (EX-1) Composite functions
I can combine functions using composition. I can find the domain of a composite function.

☐ (EX-2) Inverse functions
I can determine whether a function is one-to-one. I can obtain the graph of the inverse function from the graph of the function. I can determine the inverse of a function defined by a map or a set of ordered pairs. I can find the inverse of a function defined by an equation.

☐ (EX-3) Exponential functions
I can evaluate exponential functions. I can define the number $e$ and approximate it to five decimal places. I can graph exponential functions. I can solve basic exponential equations.

☐ (EX-4) Logarithmic functions
I can change an exponential equation to a logarithmic equation. I can change a logarithmic equation to an exponential equation. I can evaluate logarithmic expressions without using a calculator. I can determine the domain of a logarithmic function. I can graph logarithmic functions. I can solve basic logarithmic equations.

# Review Problems

1. Write each complex number in standard form $a + bi$.

   (a) $(1 - 3i) + (7 - 4i)$ (b) $(5 - 3i) - (2 + 7i)$ (c) $(5 + 3i) \cdot (2 + 7i)$

   (d) $\dfrac{1}{3 + 4i}$ (e) $\dfrac{1 + 4i}{2 - 3i}$ (f) $\dfrac{-2 \pm \sqrt{-54}}{4}$

   (g) $(2 + i)^3$

2. Find all solutions (real and complex)

   (a) $x^2 + 25 = 0$ (b) $x^2 - 4x = -8$ (c) $3x^2 + 5 = -4x$

3. Find all real solutions for $x$ and simplify your answers:

(a) $2x^2 = 6 - x$

(b) $\dfrac{x}{x-2} + \dfrac{1}{x+3} = \dfrac{2x+1}{x^2+x-6}$

(c) $x^2 + 2x - 2 = 0$

(d) $x^4 - 8x^2 - 9 = 0$

(e) $-5 = x + \sqrt{x+17}$

(f) $x^3 + 2x^2 - 3x - 6 = 0$

(g) $(2x-1)^{1/3} = 2$

(h) $x^6 + 3x^3 - 40 = 0$

(i) $7 - 4(x+2) = 3x + 5$

(j) $|4x + 7| = 19$

4. Simplify each of the following. Write any polynomials in standard form.

(a) $\dfrac{2 - \dfrac{1}{2}}{\dfrac{2}{3} + 1}$

(b) $\dfrac{1}{x - \dfrac{4}{x}}$

(c) $5x^2 - x - 3(x-2)^2$

(d) $5(x+1)^3$

(e) $\dfrac{x^3 + 8}{x + 2}$

(f) $\dfrac{x^2}{x^2 - x - 6} \cdot \dfrac{x^2 - 4}{x^6}$

(g) $\dfrac{(x-5)+7}{(x-5)(x+2)}$

5. Factor completely:

(a) $9x^2 - 25y^2$

(b) $x^4 + 3x^2 - 4$

(c) $a^3 - 27b^3$

(d) $y^3 + 2y^2 - 9y - 18$

(e) $4a^2(3x-1) - 9(3x-1)$

(f) $27x^3 + 1$

(g) $8x^2 - 14x - 15$

6. Let $f(x) = \frac{x-4}{2}$ and $g(x) = x^2 - 3$. Find and simplify:

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

(c) $g(x+h)$

(d) $f^{-1}(x)$

(e) $(g \circ g)(x)$

(f) $-f(x)$

(g) $f(-x)$

(h) $(f \cdot g)(x)$

(i) $(f - g)(x)$

7. Consider the quadratic function $f(x) = -2x^2 - 8x + 10$.

(a) Write in form $y = a(x-h)^2 + k$.

(b) Find the vertex.

(c) Find the $y$-intercept.

(d) Find any $x$-intercept(s).

8. Solve each of the following. Write your answer in interval notation.

   (a) $-7 \leq 1 - 2x < 3$ (b) $\frac{5}{4} - \frac{3x}{10} \geq \frac{x-8}{5}$ (c) $2x^2 + x > 15$

   (d) $\frac{3x-2}{x+1} \leq 2$ (e) $|2x - 13| < 5$ (f) $|x + 9| \geq 14$

9. Find an equation for each line. Write in slope-intercept form where possible.

   (a) Through the points $(-1, 4)$ and $(5, 2.$

   (b) Through the points $(3, -2)$ and $(3, 1)$

   (c) Parallel to the line $x - 2y + 6 = 0$ and containing the point $(-4, 3)$.

   (d) Parallel to the line $y = 4$ and containing the point $(5, -2)$.

   (e) Perpendicular to the line $3x + y - 7 = 0$ and containing the point $(1, -1)$.

10. Find the center and radius of each of the following circles:

    (a) $x^2 + y^2 + 6x + 8 = 0$ (b) $3x^2 + 3y^2 - 6x + 12y - 6 = 0$

11. (a) Find the $y$-intercept of the line $5x + 2y - 3 = 0$.

    (b) Find the slope of the line $5x + 2y - 3 = 0$.

12. Find the $y$-intercept, the zeros and the multiplicity of each, and sketch a graph of $f(x) = -x^3+3x^2$.

13. Find and simplify each of the following:

    (a) $3a^2 - 5ab^{-3}$ when $a = -4$ and $b = -2$

    (b) $5 - 2a^{-3}b^{2/3}$ when $a = -2$ and $b = 27$

14. Find the domain of each function and write using interval notation:

    (a) $g(x) = \frac{3x}{x^2+16}$ (b) $h(x) = \sqrt{6 - 2x}$ (c) $f(x) = \frac{1-x}{x^2-9}$

15. Simplify completely, reducing all fractions.

    (a) $8^{-4/3}$ (b) $\frac{\left(2a^{-3/2}b^3\right)^4}{ab}$ (c) $\left(\frac{16}{9}\right)^{-1/2}$

    (d) $\frac{1}{x-4} - \frac{7}{x^2-x-12}$ (e) $\sqrt{45x^{13}}$ (f) $\frac{8}{a^2+2a} - \frac{4}{a}$

    (g) $\sqrt{18} - \sqrt{2}$

16. Find the equation of a polynomial function $y = f(x)$ satisfying the following:

    Zeros are: $0$ of multiplicity $3$, $-1$ of multiplicity $2$, $3$ of multiplicity $1$

    Leading coefficient: $-1$

    Write your answer in factored form.

17. Consider the points $A(3,-2)$ and $B(5,4)$.

    (a) Find the midpoint of the line segment joining $A$ and $B$.

    (b) Find the distance between $A$ and $B$ and simplify.

18. Solve for $x$:

    (a) $4^{3-2x} = \dfrac{1}{8}$ (b) $\left(\dfrac{1}{3}\right)^{x-5} = 81$ (c) $e^{3x-4} = 7$ (d) $\log_2(6x-1) = 4$

19. A quadratic function $y = f(x)$ has vertex at $(-2,3)$ and passes through $(0,-3)$.

    (a) Find the equation of the function and write in expanded form.

    (b) Find the domain of the function in interval notation.

    (c) Find the range of the function in interval notation.

    (d) Find the $x$-intercepts.

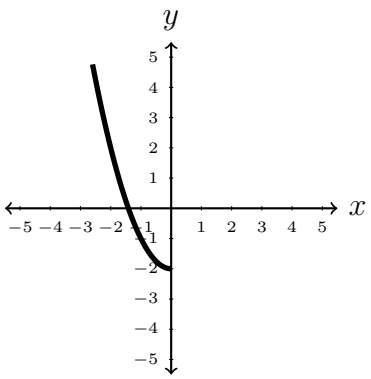
20. Compute and simplify:

    (a) $\log_5 25$ (b) $\log_2(2^{10})$ (c) $\log_7 7$ (d) $\log_8 1$ (e) $\log_3\left(\frac{1}{9}\right)$

21. Consider the function $f(x) = 2^x - 2$.

    (a) Find the $y$-intercept. (b) Find any $x$-intercepts. (c) Graph $y = f(x)$.

22. The graph of $y = f(x)$ is shown below. Draw the graph of $y = f^{-1}(x)$.




23. Solve the system of equations

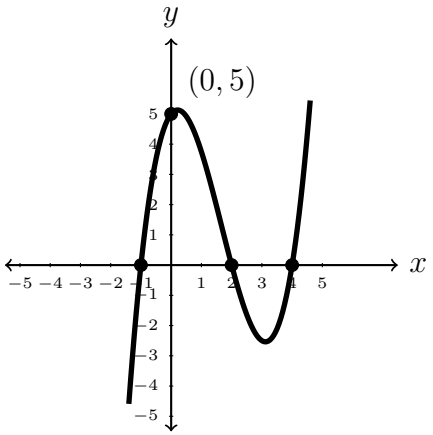
$$\begin{cases} 2x + 2y = 5 \\ 4x + \ \ y = 1 \end{cases}$$

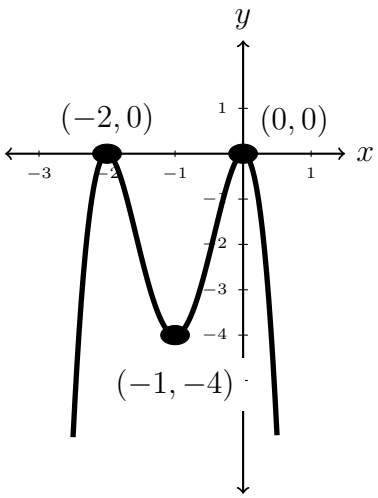
24. Solve for $a$ in the equation

$$3x = \frac{a - 2b}{k + a}$$

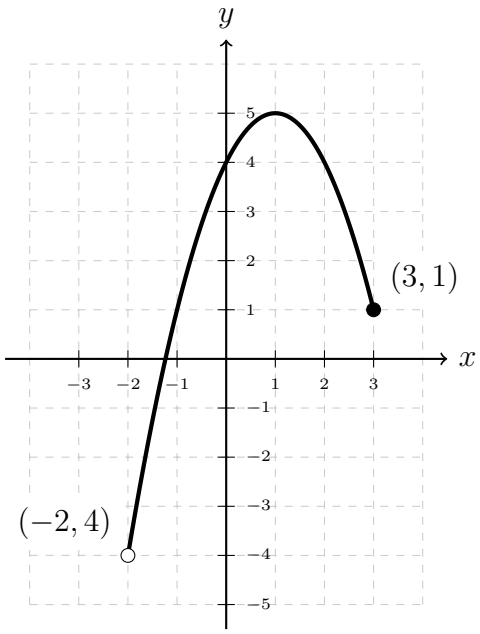
25. Find the domain of $f(x) = \sqrt{x^2 - x - 6}$.

26. Find the equations of the polynomials with lowest possible degree with the graphs shown below.




27. Use the graph of the function $f$ shown below to answer parts (a)-(e).




(a) Find $f(2)$.

(b) For what value(s) of $x$ is $f(x) = 1$?

(c) What is the domain of $f$?

(d) What is the range of $f$?

(e) How many times does the graph of $y = 4$ cross the graph of $f$?

28. For the rational functions given, find any $x$-intercepts, any $y$-intercept, equations of any vertical or horizontal asymptotes, and state the domain using interval notation.

(a) $f(x) = \dfrac{3x-2}{x+1}$ (b) $f(x) = \dfrac{x+5}{x^2-9}$ (c) $f(x) = \dfrac{x^2-3x-4}{x-2}$

29. Rationalize the denominator and simplify your answer: (a) $\dfrac{6}{\sqrt{10}}$ (b) $\dfrac{2}{5-\sqrt{3}}$

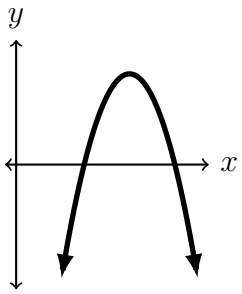
30. Find the quotient and remainder when $2x^4 - 5x^3 + x - 7$ is divided by $x^2 + 3$.

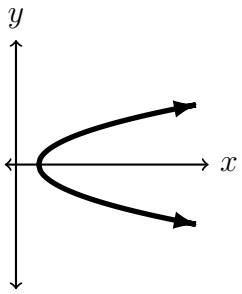
31. For each function $f(x)$, find $\dfrac{f(x+h)-f(x)}{h}$ and simplify your answer.

    (a) $f(x) = 4 - 3x$ (b) $f(x) = 5x^2 + x - 3$ (c) $f(x) = \frac{1}{x+2}$

32. Which of the following is/are functions?

    (a) $x^2 + y^2 = 9$

    (b)
    
    

    (c)
    
    

33. Consider the function $f(x) = \log_3(-x)$.

    (a) Find any $y$-intercept.

    (b) Find any $x$-intercept(s).

    (c) Graph $y = f(x)$.

# Answers

1. (a) $8 - 7i$, (b) $3 - 10i$, (c) $-11 + 41i$, (d) $\frac{3}{25} - \frac{4}{25}i$, (e) $\frac{-10}{13} + \frac{11}{13}i$, (f) $\frac{-1}{2} + \frac{3\sqrt{6}}{4}i$, (g) $2 + 11i$

2. (a) $\pm 5i$ (b) $2 \pm 2i$, (c) $-\frac{2}{3} \pm \frac{\sqrt{11}}{3}i$

3. (a) $3/2, -2$ (b) $1$ (c) $-1 \pm \sqrt{3}$ (d) $3, -3$ (e) $-8$ (f) $-2, \pm\sqrt{3}$ (g) $9/2$, (h) $-2, \sqrt[3]{5}$, (i) $-6/7$, (j) $3$, $-13/2$

4. (a) $9/10$, (b) $\frac{x}{x^2-4}$, (c) $2x^2 + 11x - 12$, (d) $5x^3 + 15x^2 + 15x + 5$, (e) $x^2 - 2x + 4$, (f) $\frac{x-2}{x^4(x-3)}$, (g) $\frac{1}{x-5}$

5. (a) $(3x-5y)(3x+5y)$, (b) $(x^2+4)(x-1)(x+1)$, (c) $(a-3b)(a^2+3ab+9b^2)$, (d) $(y+3)(y-3)(y+2)$, (e) $(2a-3)(2a+3)(3x-1)$, (f) $(3x+1)(9x^2-3x+1)$, (g) $(2x-5)(4x+3)$

6. (a) $\frac{x^2-7}{2}$, (b) $\frac{x^2-8x+4}{4}$, (c) $x^2 + 2xh + h^2 - 3$, (d) $f^{-1}(x) = 2x + 4$, (e) $x^4 - 6x^2 + 6$, (f) $\frac{-x+4}{2}$, (g) $\frac{-x-4}{2}$, (h) $\frac{x^3-4x^2-3x+12}{2}$, (i) $\frac{x-2x^2+2}{2}$

7. (a) $f(x) = -2(x+2)^2 + 18$, (b) $(-2, 18)$, (c) $(0, 10)$, (d) $(-5, 0)$, $(1, 0)$

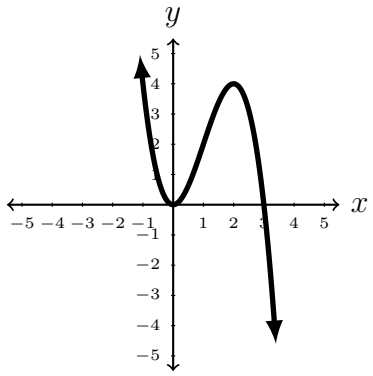
8. (a) $(-1, 4]$, (b) $\left(-\infty, \frac{57}{10}\right]$, (c) $(-\infty, -3) \cup \left(\frac{5}{2}, \infty\right)$, (d) $(-1, 4]$, (e) $(4, 9)$, (f) $(-\infty, -23] \cup [5, \infty)$

9. (a) $y = -\frac{1}{3}x + \frac{11}{3}$, (b) $x = 3$, (c) $y = \frac{1}{2}x + 5$, (d) $y = -2$, (e) $y = \frac{1}{3}x - \frac{4}{3}$

10. (a) Center $(-3, 0)$, radius $r = 1$. (b) Center $(1, -2)$, radius $\sqrt{7}$

11. (a) $(0, 3/2)$ (b) $m = -\frac{5}{2}$

12. $y$-int $(0,0)$, zero $x=0$ multiplity 2, zero $x=3$ multiplicity 1




13. (a) $91/2$, (b) $29/4$

14. (a) $(-\infty,\infty)$, (b) $(-\infty,3]$, (c) $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

15. (a) $1/16$, (b) $\frac{16b^{11}}{a^7}$, (c) $3/4$, (d) $\frac{1}{x+3}$, (e) $3x^6\sqrt{5x}$, (f) $\frac{-4}{a+2}$, (g) $2\sqrt{2}$

16. $f(x)=-x^3(x+1)^2(x-3)$

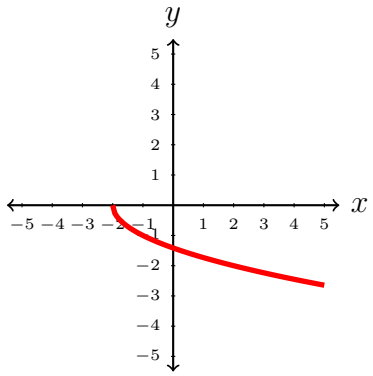
17. (a) midpoint $(4,1)$, (b) distance $2\sqrt{10}$

18. (a) $9/4$, (b) $1$, (c) $\frac{4+\ln(7)}{3}$, (d) $17/6$

19. (a) $f(x)=-\frac{3}{2}x^2-6x-3$, (b) $(-\infty,\infty)$, (c) $(-\infty,3]$, (d) $\left(-2\pm\sqrt{2},0\right)$

20. (a) $2$, (b) $10$, (c) $1$, (d) $0$, (e) $-2$

21. (a) $(0,-1)$, (b) $(1,0)$, (c) Graph

22. 


23. $x=-1/2, y=3$

24. $a=\frac{3kx+2b}{1-3x}$ or $a=\frac{-3kx-2b}{3x-1}$

25. $(-\infty,-2]\cup[3,\infty)$

26. (a) $y=\frac{5}{8}(x+1)(x-2)(x-4)$, (b) $y=-4x^2(x+2)^2$

27. (a) $f(2)=4$, (b) $x=-1$, $x=3$, (c) $-2<x\leq 3$, (d) $4<y\leq 5$, (e) Twice

28. (a) $x$-int $(2/3,0)$, $y$-int $(0,-2)$, VA $x=-1$, HA $y=3$, domain $(-\infty,-1)\cup(-1,\infty)$

(b) $x$-int $(-5, 0)$, $y$-int $(0, -5/9)$, VA $x = -3$ and $x = 3$, HA $y = 0$, domain $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

(c) $x$-int $(-1, 0)$ and $(4, 0)$, $y$-int $(0, 2)$, VA $x = 2$, HA None, domain $(-\infty, 2) \cup (2, \infty)$

29. (a) $\frac{3\sqrt{10}}{5}$, (b) $\frac{5+\sqrt{3}}{11}$

30. Quotient $2x^2 - 5x - 6$, remainder $16x + 11$

31. (a) $-3$, (b) $10x + 5h + 1$, (c) $\frac{-1}{(x+h+2)(x+2)}$

32. (a) not a function, (b) function, (c) not a function

33. (a) None, (b) $(-1, 0)$, (c)