Modeling a two-phase gas-liquid flow for mucus transport in human lungs

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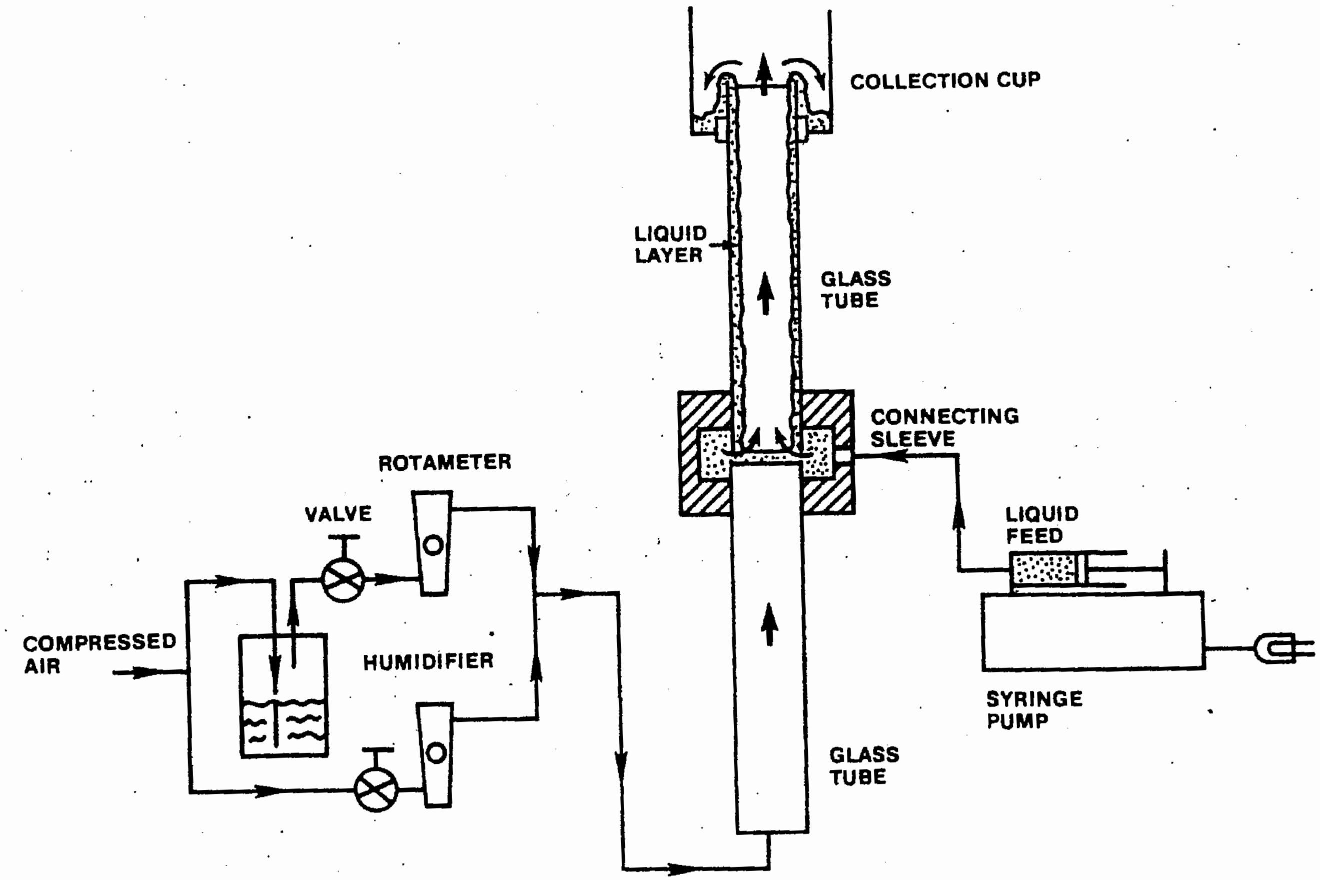
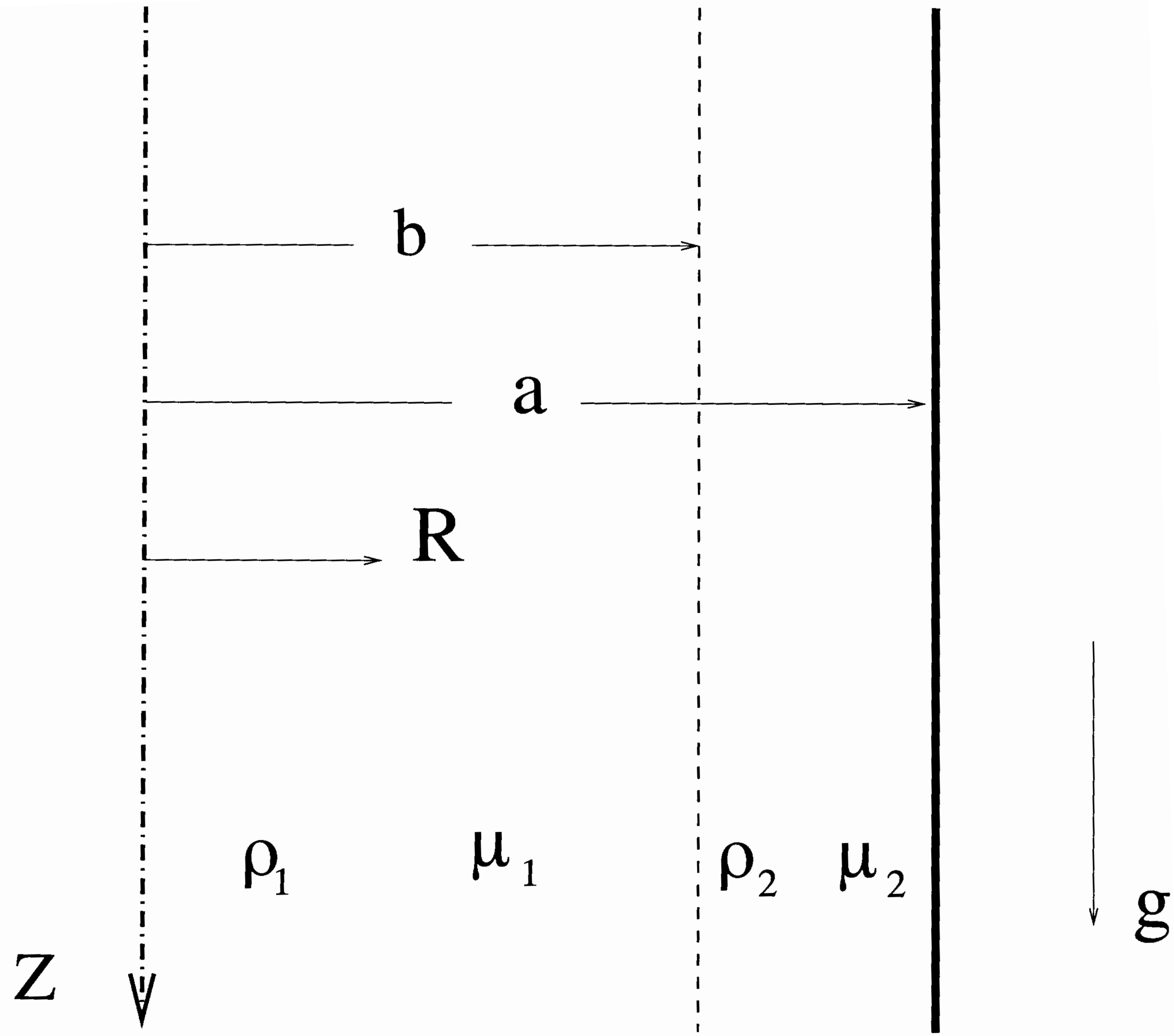


FIG. 1. Schematic diagram of experimental system showing a vertical tube-flow model.

C. S. Kim, M A. Greene, S. Sankaran,
and M. A. Sackner

The American Physiological Society, J. Apply. Physiol.



b
a
R
ρ1
μ 1
ρ2
μ 2
g
Z

The Navier-Stokes & continuity equations (in cylindrical coordinates (R, θ, z))

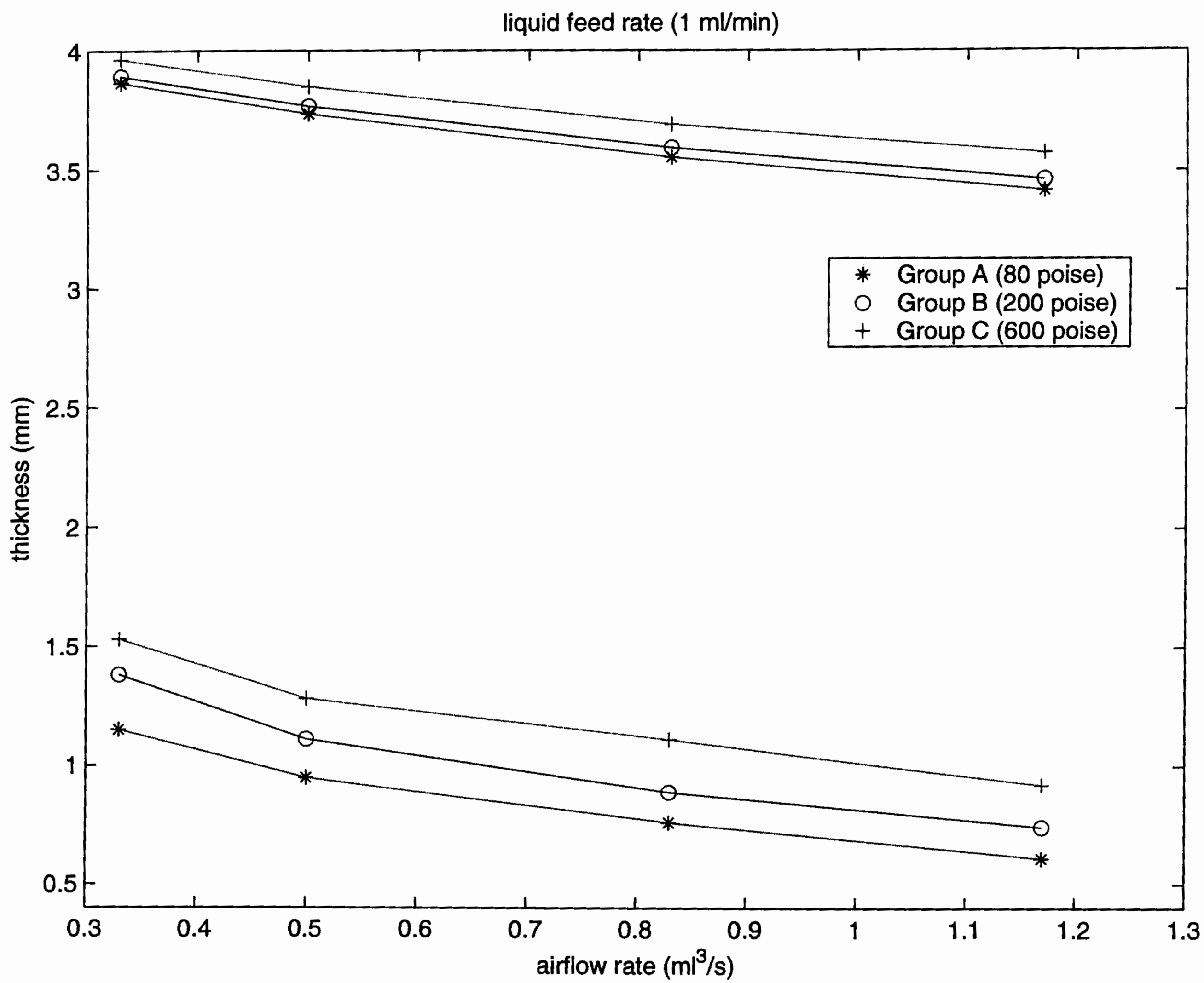
$$\underset{\sim}{u}_t + (\underset{\sim}{u} \cdot \nabla)\underset{\sim}{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \underset{\sim}{u} - g\hat{e}_z$$

$$\nabla \cdot \underset{\sim}{u} = 0$$

The primary flow:

$$\begin{cases} W^{(l)} = \dfrac{K_1}{4\mu_1} R^2 + B \ln R + E \\ W^{(a)} = \dfrac{K_2}{4\mu_2} R^2 + D \end{cases}$$

5 unknowns, 5 boundary (interfacial) conditions



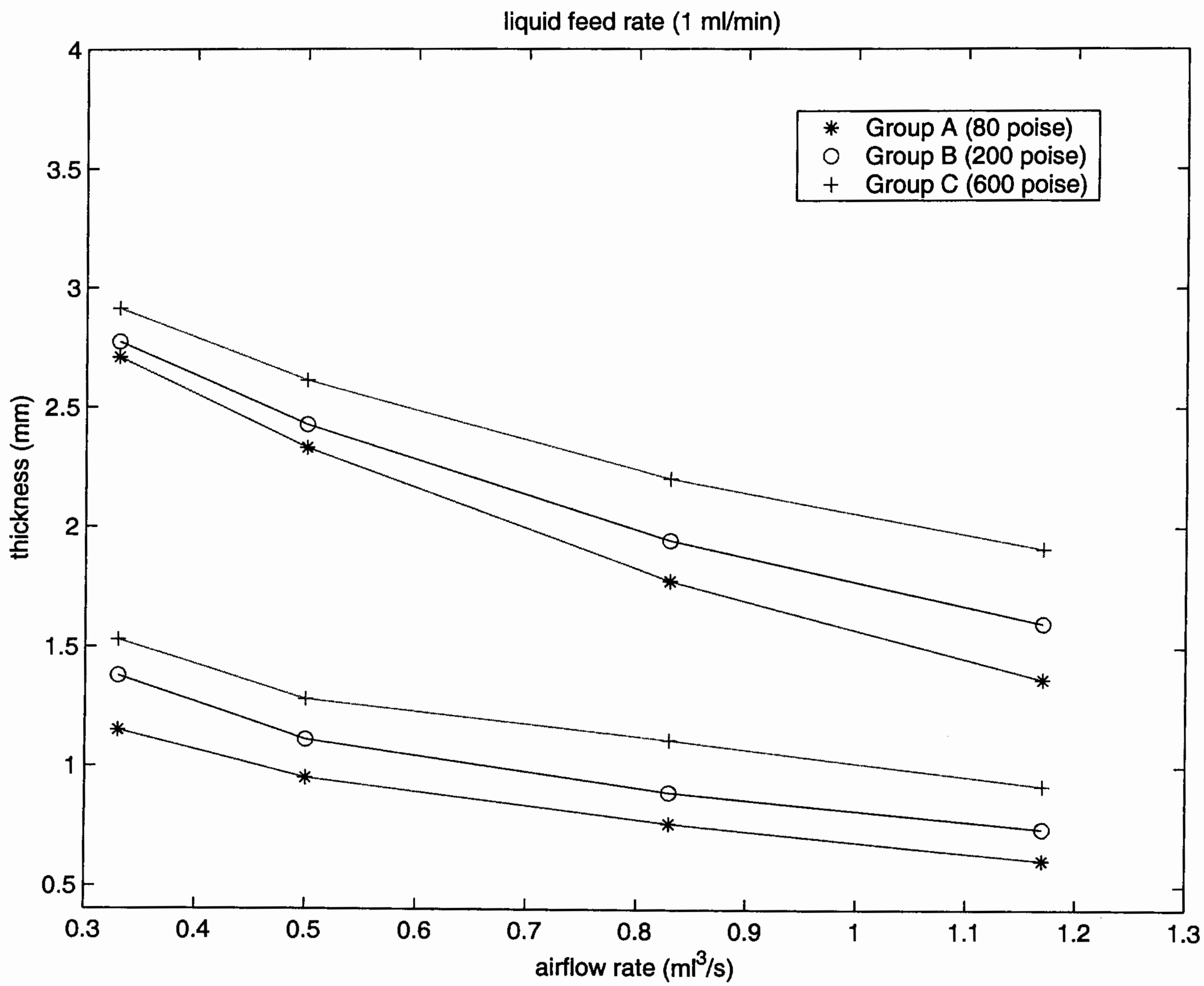
liquid feed rate (1 ml/min)
Group A (80 poise)
Group B (200 poise)
Group C (600 poise)
thickness (mm)
airflow rate (ml^3/s)
4
3.5
3
2.5
2
1.5
1
0.5
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3

The effective viscosity (D. D. Joseph et. al. 1996)

$$\mu_{T.G} \simeq 0.016 \rho_G \frac{J_G D}{16}$$

J_G is the superficial velocity ~ mean velocity

D is the diameter of the tube.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Stability Analysis (C.E. Hickox, JFM. 14. 1971)

Assumed that the primary flow is disturbed slightly.
the inner core region

$$u = u', \quad v = v', \quad w = W + w', \quad p = \bar{p} + p'$$

where the perturbation terms are assumed of forms

$$\{u', v', w'\} = w_1(0)\{iG(r), H(r), F(r)\} \cdot \exp[in\theta + i\alpha(z - c\tau)]$$

$$\frac{p'}{\rho} = w_1(0)\, P(r) \exp[in\theta + i\alpha(z - c\tau)]$$

$n \neq 0$ Asymmetric (angular disturbance), Axisymmetric Otherwise

Wave speed c, can be complex, $Im(c)$ determines stability.

- Application of the disturbed primary flow to the Navir-Stokes equation.

  8 equations, 8 unknowns (4 each layer)

- 12 undetermed constants giving arise from the 8 differential equations (ODEs' in r)

- 12 boundary and interfacical conditions.

- A method of regular perturbation around $\alpha = 0$, where α is the wave number.

- The quantity $\alpha Re \ll 1$. (For any Re, $\exists$ a α such that the perturbation is valid)

- The unknowns and C are expanded in series as

$$[G, H, P] = \sum_{j=1}^{N} \alpha^j [G_j, H_j, P_j]$$

$$[F, c] = \sum_{j=0}^{N} \alpha^j [F_j, c_j]$$

- The non-dimensional quantities emerge from the process are $m = \dfrac{\mu_2}{\mu_1}$, $l = \dfrac{\rho_2}{\rho_1}$.

$$R_e = \frac{W_1(0)\, b\, \rho_1}{\mu_1}, \quad S = \frac{T}{\rho_1 b W_1^2(0)}$$

[$(l/m) R_e$ is the Reynolds number for the liquid layer]

- For both $n=0$ and $n=1$, the first approximation produce real C_0, no instablility will be manifested.
- Next approximation is needed for both asymmetric and axisymmetic disturbance to observe the mode of instability.

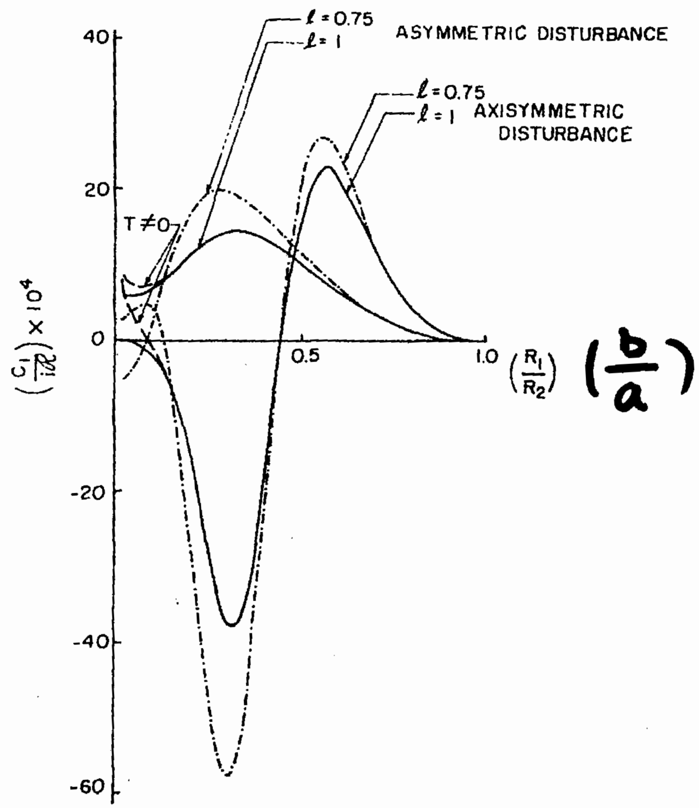


FIG. 3. Effect of the density ratio l and surface tension T in unidirectional flow with $m = 20$. For $T \neq 0$, the pressure gradient is 60 lb/ft^3 and $T = 0.0035$ lb/ft, elsewhere the pressure gradient is 30 lb/ft^3.

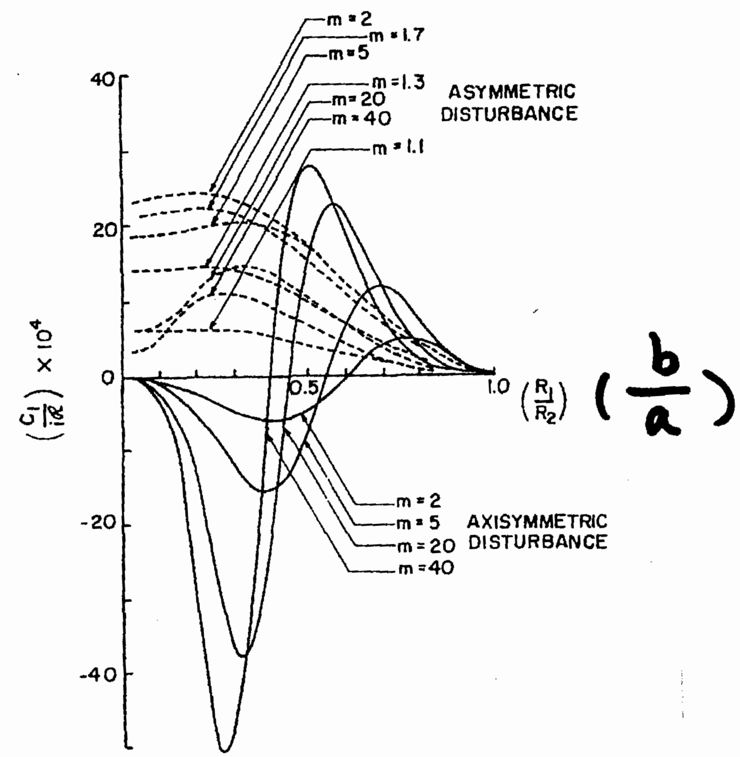
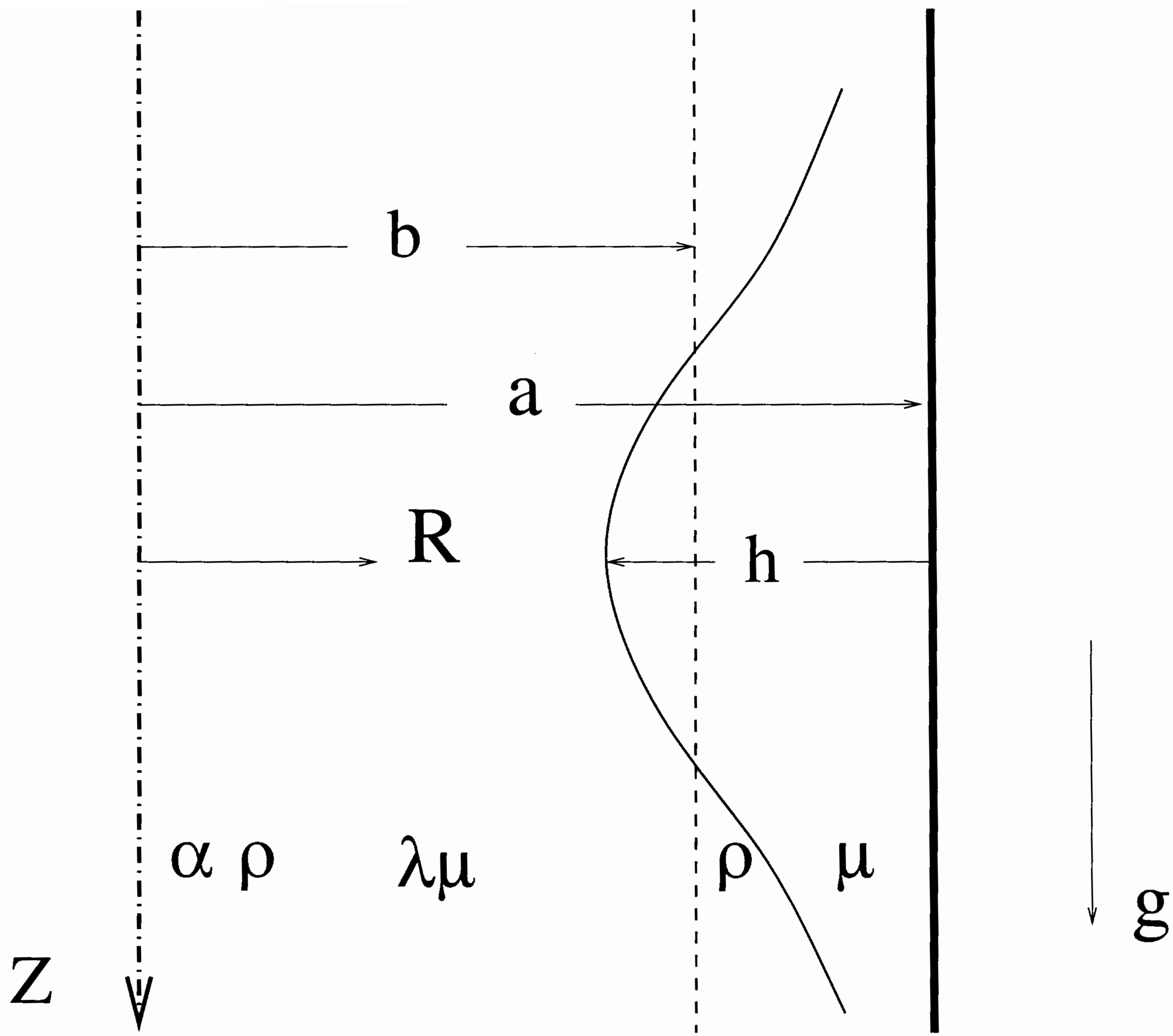


FIG. 2. Effect of the viscosity ratio m in unidirectional flow with $l = 1$, $T = 0$, and $k = 1$.

C. E Hickox

The physics of fluids V 14, No 2 P251, 1971



b
a
R
h
α ρ
λμ
ρ
μ
g
Z

Interfacial waves (P.S. Hammod, JFM 137) 1983

- Stokes flow for both inner & outer core.
- Thin film assumption

$$\varepsilon = \frac{a-b}{a} \ll 1$$

- After orders of magnitude analysis velocity field and pressure are scaled according to the surface tension and ε.
- Applications of boundary & interfacial conditions gives the evolution equation of H. (nondimensional h)

$$H_t = -\frac{1}{3}\left(H^3\left(H_{zzz} + H_z\right)\right)_z$$

If one seek separable solutions of form

$$H = H_0 \exp(ikx + st)$$

which is a complet set of "normal modes"

$$s \simeq k^2 - k^4$$

Note that: $-H_{zz}$ does not show up in the Planary case.